The antifield-BRST approach to (gauge) field theories: an overview

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Exceptional Dimensions
A Conference in Honour of Bernard Julia
(16-17 December 2019, Institut Henri Poincaré)

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- Conclusions

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This different point of view turns out to be crucial for computing the BRST cohomology.

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[For local objects, jet space formalism can be used to put these considerations on a firmer footing.]

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Nilpotency of s implies $\delta^2 = 0$, $\delta\gamma + \gamma\delta = 0$, $\gamma^2 = 0$.

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We shall for this reason only focus here on the Koszul-Tate differential δ.

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$$h = k_\mu^a A_a^{*\mu} + k_\mu^{a\rho} \partial_\rho A_a^{*\mu} + k_\mu^{a\rho\sigma} \partial_\sigma \partial_\rho A_a^{*\mu} + \cdots$$

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Thus, the Koszul-Tate complex provides a resolution of the algebra $C^\infty(\Sigma)$ of smooth functions on the stationary surface.

(If one includes the ghosts, one gets $C^\infty(\Sigma) \otimes \Lambda(C^a, \partial_\mu C^a, \cdots)$.)

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(2) When the gauge transformations are “open" (on-shell closure only"), the construction is more elaborate because $\gamma^2 \neq 0$, but $\gamma^2 \approx 0$ (only on-shell). This requires additional terms in s,

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This is the Batalin-Vilkovisky construction, which works because the Koszul-Tate complex is a resolution.

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Procedure works and corresponding term in the solution of the master equation is $\sim \int d^n x C^{*\mu} \partial_\mu C$.

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The BRST differential $s = \delta + \gamma + \cdots$ can be constructed as before

but is not generated in the antibracket through the solution S of the master equation.

Local Functionals

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So a local functional is

$$F = \int \omega, \quad \omega = f d^n x,$$

where f is a local function.

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This defines the mod-d cohomology $H^m(s|d)$.

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For instance an abelian ghost C^* fulfills $\delta C^* = \partial_\mu A^{*\mu}$ and defines a non-trivial element of $H_2(\delta|d)$.

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Reading the cocycle condition $\delta\omega + da = 0$ in both directions, one easily proves the isomorphisms

$$H^i_j(\delta|d) \simeq H^{i-1}_{j-1}(d|\delta), \ i,j > 1, \ (i,j) \neq (1,1); \ H^1_1(\delta|d) \simeq \frac{H^0_0(d|\delta)}{\mathbb{R}}$$

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Once this is done, one can compute explicitly the BRST cohomology $H(s|d)$ (BRST cohomology in the space of local functionals).

The understanding that s involves δ – and the corresponding spectral sequence – is crucial for this purpose.

Conclusions and comments

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This point of view turns out to be crucial for computing the BRST cohomology.

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THANK YOU !

Backup Material

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