bla

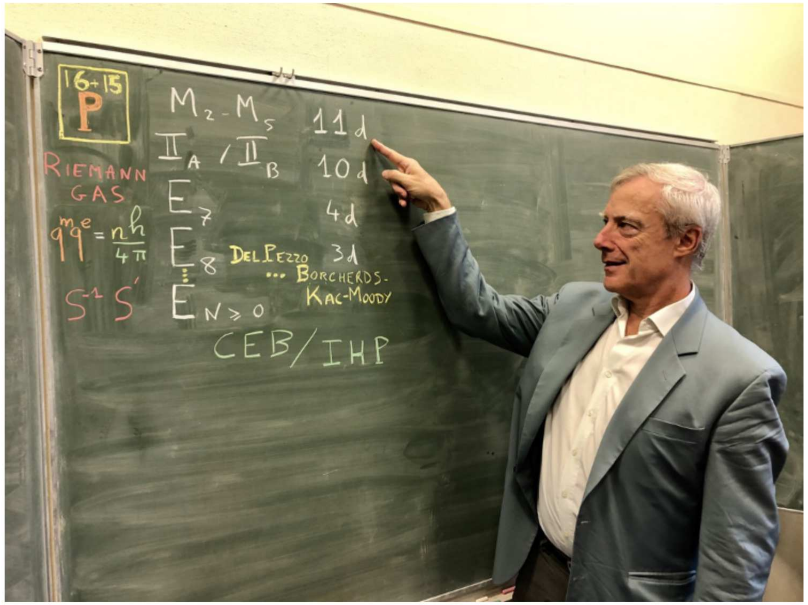
$N = 8$ Supergravity and the Real World

En l'honneur de Bernard Julia

Institut Henri Poincaré, 17 December 2019

Hermann Nicolai
MPI für Gravitationsphysik, Potsdam
(Albert Einstein Institut)

Towering Achievements



- **Maximal supergravity** [Cremmer,Julia,Scherk(1978);Cremmer,Julia(1979)]
- **Hidden exceptional duality symmetries** [Cremmer,Julia(1979)]
- **Emergence of E_{10} in reduction to $D = 1$?** [Julia(1982)]
- **... as well as many others**

$N = 8$ Supergravity

Unique theory (modulo gauging), *most symmetric* known field theoretic extension of Einstein's theory

$$\mathbf{1} \times [2] \oplus \mathbf{8} \times \left[\frac{3}{2}\right] \oplus \mathbf{28} \times [1] \oplus \mathbf{56} \times \left[\frac{1}{2}\right] \oplus \mathbf{70} \times [0]$$

$\rightarrow$ **descends from D=11 SUGRA** [Cremmer,Julia,Scherk(1978)]

In the late 1970s this theory was thought to be a promising candidate for a unified theory of quantum gravity and matter interactions. However,

- **Question of UV finiteness (or not)?**
- **Phenomenology (chiral fermions, SUSY breaking, huge negative cosmological constant,...)?**

Nevertheless: large part of work since 1980s on string unification is really based on, or inspired by maximal supergravity and its hidden symmetries E_7, E_8 ,...!

Finiteness: to be or not to be?

We now know that $N=8$ supergravity is more finite than expected: behaves like $N=4$ super-Yang-Mills up to four loops [Bern,Carrasco,Dixon,Johansson, Roiban, PRL103(2009)081301]

- **However: recent computation at five loops shows divergence at $D = \frac{24}{5} = 2 + \frac{14}{L} < \frac{26}{5} = 4 + \frac{6}{L}$ (for $L = 5$)** [Bern,Carrasco,Chen,Edison,Johansson,Parra-Martinez,Roiban,PRD98(2018)086021]

Thus: question of finiteness is still up in the air →
Although no fully supersymmetric and fully $E_{7(7)}$ invariant counterterm known, finiteness would probably still require novel (so far hidden) symmetries...

But even if $N=8$ Supergravity is finite to all orders:

- what about *non-perturbative* quantum gravity?
- is there any relation to *real physics*?

Phenomenology: early (failed) attempts

1. **Focus on vector-like $SU(3) \times U(1) \subset SO(8)$, with identifications $SU(3) \equiv SU(3)_c$ and $U(1) \equiv U(1)_{em}$** [Gell-Mann(1978)]
   $\rightarrow$ does not work: color sextets and octets

2. **Following a suggestion by Cremmer and Julia: elevate (chiral) R symmetry SU(8) to a *dynamical* symmetry $\rightarrow 3 \times (\bar{5} \oplus 10)$ fermions of SU(5) GUT + much more** [Ellis,Gaillard,Zumino(1981)]

3. **Or: unitary irreps of $E_{7(7)}$?** [Ellis,Gaillard,Günaydin,Zumino(1982)]

Main problem with 2. and 3.: too much junk! (much like for low energy SUSY/MSSM model building...)

Prevailing view (since about 1982): $N=8$ supergravity is *obviously not* a good candidate for quantum gravity and the unification of all interactions!

However: $\mathbf{56 - 8 = 48 = 3 \times 16}$ **!**

A strange coincidence?

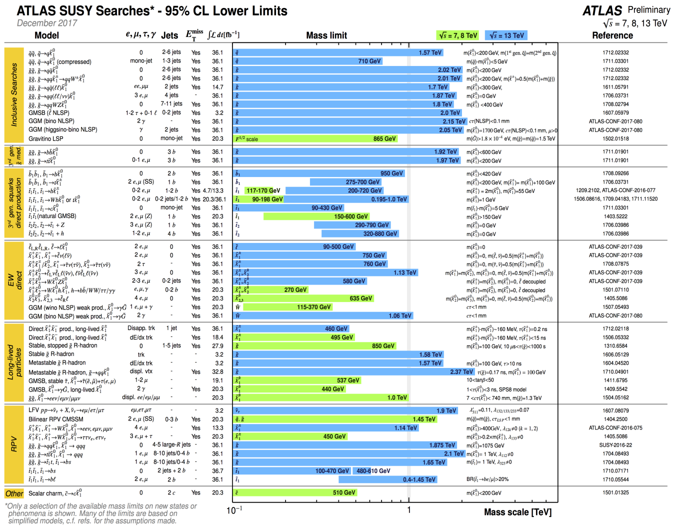
$SO(8) \to SU(3)\times U(1)$ **breaking and 'family-color locking'**

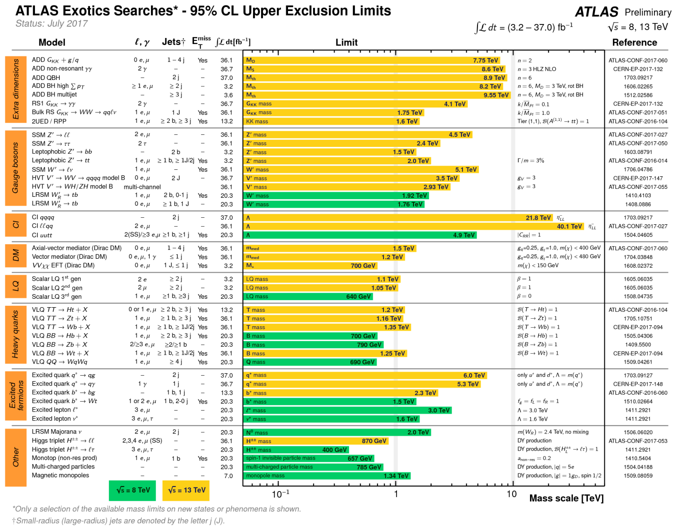
$$
\begin{array}{llll}
(u\,,\,c\,,\,t)_L & : & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,, & +\frac{1}{2} = \frac{2}{3} - q \\
(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L & : & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,, & -\frac{1}{2} = -\frac{2}{3} + q \\
(d\,,\,s\,,\,b)_L & : & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,, & -\frac{1}{6} = -\frac{1}{3} + q \\
(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L & : & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,, & +\frac{1}{6} = \frac{1}{3} - q \\
(e^-, \mu^-, \tau^-)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & -\frac{5}{6} = -1 + q \\
(e^+, \mu^+, \tau^+)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & +\frac{5}{6} = 1 - q \\
(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & -\frac{1}{6} = 0 - q \\
(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & +\frac{1}{6} = 0 + q
\end{array}
$$

Supergravity and Standard Model assignments agree if spurion charge is chosen as $q = \frac{1}{6}$ [Gell-Mann (1983)]

Realized at $SU(3)\times U(1)$ **stationary point!** [Warner,HN, NPB259(1985)412]

(No) News from LHC





Exclusion limits, nothing but exclusion limits, ...

- No hints whatsoever of new physics
- RG Evolution of (slightly amended) SM couplings: no Landau poles, no instabilities of effective potential up to Planck scale

Conclusion (so far, at least): SM could survive more or less *as is* all the way to Planck scale M_{PL}!

Fixing the U(1) mismatch

[Meissner,HN: Phys.Rev.D91(2015)065029]

Spurion charge shift can be realised as $\exp(\frac{1}{6}\omega\mathcal{I})$

$$\mathcal{I} = \frac{1}{2}\big(T\wedge\mathbf{1}\wedge\mathbf{1} + \mathbf{1}\wedge T\wedge\mathbf{1} + \mathbf{1}\wedge\mathbf{1}\wedge T + T\wedge T\wedge T\big) \quad\Rightarrow\quad \mathcal{I}^2 = -\mathbf{1}$$

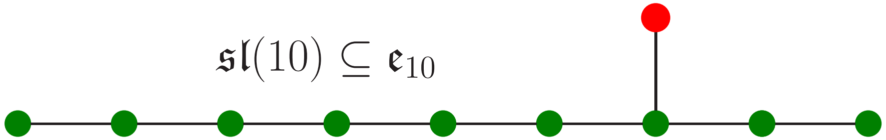
acting on 56 fermions χ^{ijk} **in** $\mathbf{8}\wedge\mathbf{8}\wedge\mathbf{8}$ **of SU(8), with**

$$T = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \qquad T^2 = -\mathbf{1}$$

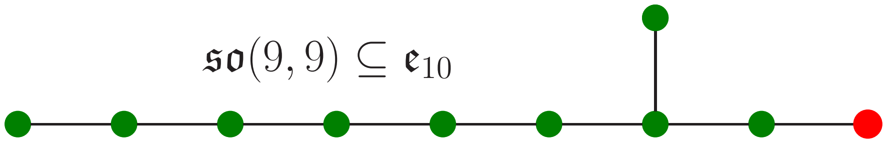
However: $\mathcal{I}$ **is *not* in SU(8)** $\equiv K(\mathrm{E}_7) \quad\Rightarrow$
mismatch can *not* be fixed *within* $N=8$ **supergravity.**

Claim: to accommodate deformed U(1) we have to go all the way to $\mathrm{K}(\mathrm{E}_{10})$ **(and thus** E_{10}**)!**

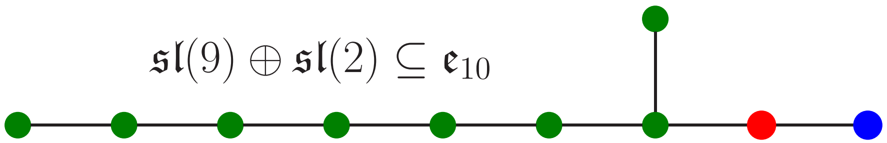
Duality symmetries: all in one ($= \mathrm{E}_{10}$)?



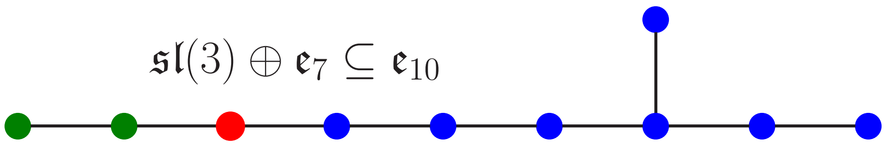
$D = 11$ **SUGRA**



mIIA $D = 10$ **SUGRA**



IIB $D = 10$ **SUGRA**



$\mathcal{N} = 8,\ D = 4$ **SUGRA**

Fermions and $\mathrm{K}(\mathrm{E}_{10})$

... probably a key issue for further progress...

Important point: maximally supersymmetric theories *not* based on (hypothetical) superextensions of E_n:

- **There is no proper superextension of E_n for any n.**
- **For $D \geq 3$ supergravity fermions transform in *maximal compact subgroup* $K(E_n) \subset E_{n(n)}$, e.g.**

$$K(E_7) \equiv SU(8) \qquad \text{fermions} \in \mathbf{8} \text{ and } \mathbf{56}$$

$$K(E_8) \equiv Spin(16)/Z_2 \qquad \text{fermions} \in \mathbf{16}_v \text{ and } \mathbf{128}_c$$

- **The associated (double-valued) fermion representations are not 'liftable' to E_n representations**
- **Expect all of this to remain true for $\mathrm{K}(\mathrm{E}_{10}) \subset \mathrm{E}_{10}$.**

What is $\mathrm{K}(\mathrm{E}_{10})$?

For E_{10}, the 'maximal compact' subalgebra is defined as the fixed point algebra of the Chevalley involution

$$\omega(e_j) = -f_j\ , \quad \omega(f_j) = -e_j\ , \quad \omega(h_j) = -h_j$$

together with invariance property $[\omega(x), \omega(y)] = \omega([x, y])$

$$\Rightarrow \quad \mathrm{E}_{10} = K(\mathrm{E}_{10}) \oplus K(\mathrm{E}_{10})^{\perp}\ , \qquad x = \omega(x) \text{ for } x \in K(\mathrm{E}_{10})$$

This definition is analogous to the corresponding one for the finite-dimensional case, e.g. $x = \omega(x) \in \mathfrak{so}(n) \subset \mathfrak{sl}(n)$ **for** $\omega(x) = -x^T$**, with corresponding decomposition** $\mathfrak{sl}(n) = \mathfrak{so}(n) \oplus \mathfrak{so}(n)^{\perp}$

Consequently, $\mathrm{K}(\mathrm{E}_{10})$ **is generated by**

$$x_i := e_i - f_i = \omega(x_i) \qquad i, j, ... = 1, ..., 10$$

with Berman-Serre relations (for E_{10} Dynkin diagram)

$$\left[x_i\,,\,x_j\right] \;=\; 0 \qquad \textit{if } i \textit{ and } j \textit{ are non-adjacent}$$

$$\left[x_i,[x_i,x_j]\right] + x_j \;=\; 0 \qquad \textit{if } i \textit{ and } j \textit{ are adjacent}$$

Theorem: ***each set of*** $\{x_i\}$ ***satisfying the above relations provides a realization of*** $\mathrm{K}(\mathrm{E}_{10})$***.*** [S.Berman(1989)]

Involutory subalgebra $\mathrm{K}(\mathrm{E}_{10}) \subset \mathrm{E}_{10}$ **is spanned by** $\{J^r_\alpha\}$

$$J^r_\alpha \equiv E^r_\alpha - E^r_{-\alpha}\,, \qquad \alpha \in \Delta_+(\mathrm{E}_{10})\,, \qquad r = 1, ..., \mathrm{mult}(\alpha)$$

But: $K(\mathrm{E}_{10})$ **is ∞-dimensional and a very strange beast!**

- $\mathrm{K}(\mathrm{E}_{10})$ **is *not* a Kac–Moody algebra** [Kleinschmidt,HN: CQG22(2005)4457]
- $\mathrm{K}(\mathrm{E}_{10})$ **has finite-dimensional (unfaithful) representations**
- $\Rightarrow$ $\mathrm{K}(\mathrm{E}_{10})$ **is *not* simple ($\equiv$ has non-trivial ideals)**
- **No faithful (infinite-dimensional) representations are known**

Unfaithful representations

$\iff$ **existence of non-trivial ideals $\mathfrak{i}_V$ in $K(\mathrm{E}_{10})$!**

More precisely: for unfaithful representation V the associated ideal is

$$\mathfrak{i}_V := \left\{ x \in \mathrm{K}(\mathrm{E}_{10}) \,\middle|\, x \cdot v = 0 \ \forall v \in V \right\} \subset \mathrm{K}(\mathrm{E}_{10})$$

For known examples, $\mathfrak{i}_V$ has *finite* co-dimension in $\mathrm{K}(\mathrm{E}_{10})$

$\Rightarrow$ $\mathfrak{i}_V^{\perp} \equiv \mathrm{K}(\mathrm{E}_{10}) \ominus \mathfrak{i}_V$ **is *not* a subalgebra of $\mathrm{K}(\mathrm{E}_{10})$.**

(is spanned by non-convergent sums) [Kleinschmidt,Palmkvist,HN:JHEP(2007)051]

Analysis of fermionic sector of $D\!=\!11$ SUGRA $\Rightarrow$

Spin-$\frac{1}{2}$ ('Dirac representation' $V_{1/2}$): [deBuyl,Henneaux,Paulot(2005)]

$$J^{(0)}_{ab}\chi = \frac{1}{2}\Gamma_{ab}\chi, \qquad J^{(1)}_{abc}\chi = \frac{1}{2}\Gamma_{abc}\chi$$

Spin-$\frac{3}{2}$ ('Rarita-Schwinger representation' $V_{3/2}$) [DKN,dBHP(2006)]

$$J^{(0)}_{ab}\psi_c = \frac{1}{2}\Gamma_{ab}\psi_c + 2\delta_c^{[a}\psi^{b]}\ , \qquad J^{(1)}_{abc}\psi_d = \frac{1}{2}\Gamma_{abc}\psi_d + 4\delta_d^{[a}\Gamma^{b}\psi^{c]} - \Gamma_d{}^{[ab}\psi^{c]}.$$

Multiple commutators generate full $\mathrm{K}(\mathrm{E}_{10})$ algebra:

$$\left[J^{(1)}_{abc}\,,\;J^{(1)}_{def}\right] = J^{(2)}_{abcdef} + \delta^{[de}_{[ab}J^{(0)\,f]}_{c]} \qquad etc.$$

Rarita-Schwinger equation can be reformulated as a '$K(\mathrm{E}_{10})$ covariant Dirac equation'. [Damour,Kleinschmidt,HN(2006)]

Spin-$\frac{3}{2}$ representation contains IIA and IIB fermions, respectively, upon decomposition under corresponding (finite-dimensional) subgroups of $\mathrm{K}(\mathrm{E}_{10})$ [Kleinschmidt,HN(2006)]

More specifically: *Rarita-Schwinger* representation $\rightarrow$ 8 gravitinos and 56 spin-$\frac{1}{2}$ fermions of maximal $N=8$ supergravity *at one spatial point* form an unfaithful irreducible spinorial representation of $\mathrm{K}(\mathrm{E}_{10})$.

Fermionic structure of $N=8$ supermultiplet can thus be viewed as a consequence of $\mathrm{K}(\mathrm{E}_{10})$ rather than supersymmetry!

Idem for 8 *massive* gravitinos and 48 spin-$\frac{1}{2}$ fermions

Why $\mathcal{I}$ belongs to $\mathrm{K(E_{10})}$

[Kleinschmidt,HN:Phys.Lett.B747 (2015)]

D=11 fermions in Coulomb gauge split as ($\hat{a} = 1,2,3\,;\ \bar{a} = 4,...,10$)

$$\Psi_A^a = (\Psi_{\alpha i}^{\hat{a}}\,,\ \Psi_{\alpha i}^{\bar{a}}) \quad \textbf{with } i,j = 1,...,8 \textbf{ and } \alpha = 1,2,3,4$$

$N\!=\!8$ **supergravity fermions from** $D\!=\!11$ **gravitino** [Cremmer,Julia(1979)]

$$\psi_{\hat{a}\alpha}^i \propto \Psi_{\alpha i}^{\hat{a}} - \frac{1}{2}\sum_{\bar{c}=4}^{10} \Gamma_{ij}^{\bar{c}}(\gamma^5\gamma_{\hat{a}}\Psi_j^{\bar{c}})_\alpha \quad , \qquad \chi^{ijk} \propto \sum_{\bar{a}=4}^{10} \Gamma_{[ij}^{\bar{a}}\Psi_{k]\alpha}^{\bar{a}}$$

With redefined variables $\Phi_A^{\mathsf{a}} = \Gamma_{AB}^a \Psi_B^a$ **(no summation!)** [Damour,Hillmann]

$$\delta\chi_{ijk} = (T \wedge T \wedge T)_{ijk}{}^{lmn}\chi_{lmn} \quad \leftrightarrow \quad \delta\Phi_{i\alpha}^{\mathsf{a}} = T_{ij}\Phi_{j\alpha}^{\mathsf{a}} \qquad (*)$$

Latter formula provides a realization of $\mathcal{I}$ on *all* fermions.

For any *real* E_{10} **root** α **we have (with** $\alpha^{\mathsf{a}} \equiv G^{\mathsf{ab}}\alpha_{\mathsf{b}}$**)** [Kleinschmidt,HN]

$$\delta(\alpha)\Phi_A^{\mathsf{a}} = \left(-\frac{1}{2}\alpha^{\mathsf{a}}\alpha_{\mathsf{b}} + \frac{1}{4}\delta_{\mathsf{b}}^{\mathsf{a}}\right)\Gamma(\alpha)_{AB}\Phi_B^{\mathsf{b}}$$

Thus need only find linear combination to reproduce $(*)$**, which is possible because there are *infinitely many* real roots in** E_{10}**.**

The proof requires over-extended root of $\mathrm{E}_{10} \Rightarrow$ no way to realise q-shift with finite-dimensional R symmetries!

More properly, this representation is acted on by

$$\mathcal{Q}_{3/2} = \mathrm{K}(\mathrm{E}_{10})/\mathcal{N}_{3/2} = \mathrm{SO}(32, 288)$$

where $\mathcal{N}_{3/2}$ is the 'normal subgroup' generated by the RS ideal in $\mathrm{K}(\mathrm{E}_{10})$ – but $\mathcal{Q}_{3/2}$ is *not* a subgroup of $\mathrm{K}(\mathrm{E}_{10})$.

In recent work we have been able to embed full SM group $\mathbf{SU(3)}_c \times \mathbf{SU(2)}_w \times \mathbf{U(1)}_Y$ into $\mathcal{Q}_{3/2}$ together with a family symmetry $\mathbf{SU(3)}_f$ which does *not* commute with electroweak symmetries. [Meissner,HN, PRL121(2018)091601]

Big open questions: how does $\mathrm{K}(\mathrm{E}_{10})$ 'unfold' to give rise to spatial dependence and space-time symmetries?

And why and how is $\mathrm{K}(\mathrm{E}_{10})$ broken to SM symmetries?

Higher spin realizations of $K(\mathrm{E}_{10})$

$\rightarrow$ **trying to break out of the confines of supergravity!**

But first need to re-write spin-$\frac{3}{2}$ by means of crucial redefinition [Damour,Hillmann:0906.3116]

$$\phi_A^{\mathsf{a}} \equiv \sum_{B=1}^{32} \Gamma^a_{AB} \psi^a_B \quad \textbf{(no sum on } a\textbf{!)}$$

Re-definition breaks manifest Lorentz symmetry, but:

$$\{\psi^a_A\,,\,\psi^b_B\}_{\text{Dirac}} = \delta^{ab}\delta_{AB} - \frac{1}{9}(\Gamma^a\Gamma^b)_{AB} \quad \Rightarrow \quad \{\phi^{\mathsf{a}}_A\,,\,\phi^{\mathsf{b}}_B\} = G^{\mathsf{ab}}\delta_{AB}$$

$\Rightarrow$ **manifest $SO(1,9)$ = invariance group of mini-superspace WDW Hamiltonian with DeWitt metric G_{ab} instead!**

From analysis of known $\mathrm{K(E_{10})}$ transformation acting in RS representation we extract a *second quantised realisation* of $\hat{J}(\alpha)$ *for all real roots* $\alpha \in \Delta(\mathrm{E}_{10})$:

$$\hat{J}(\alpha) = \left(-\frac{1}{2}\alpha_{\mathtt{a}}\alpha_{\mathtt{b}} + \frac{1}{4}G_{\mathtt{ab}}\right)\phi^{\mathtt{a}}\Gamma(\alpha)\phi^{\mathtt{b}} \qquad \forall \textbf{ roots obeying } \alpha^2 = 2$$

***New* realization with 'spin-$\frac{5}{2}$' fermions** [Kleinschmidt,HN.:1307.0413]

$$\{\phi_A^{\mathtt{ab}}\,,\,\phi_B^{\mathtt{cd}}\} = G^{\mathtt{a(c}}G^{\mathtt{d)b}}\delta_{AB} \qquad (\phi_A^{\mathtt{ab}} = \phi_A^{\mathtt{ba}})$$

Berman-Serre relations are satisfied on $\mathcal{F}$ with

$$\hat{J}(\alpha) = X(\alpha)_{\mathtt{ab\,cd}}\,\phi^{\mathtt{ab}}\Gamma(\alpha)\phi^{\mathtt{cd}}$$

and

$$X(\alpha)_{\mathtt{ab\,cd}} = \frac{1}{2}\alpha_{\mathtt{a}}\alpha_{\mathtt{b}}\alpha_{\mathtt{c}}\alpha_{\mathtt{d}} - \alpha_{(\mathtt{a}}G_{\mathtt{b})(\mathtt{c}}\alpha_{\mathtt{d})} + \frac{1}{4}G_{\mathtt{a(c}}G_{\mathtt{d)b}}$$

again *for all* real roots α. Could also be coupled to $E_{10}/K(E_{10})$ sigma model to go beyond supergravity!

Similar ansatz also works for for spin-$\frac{7}{2}$ fermions:

$$\{\phi_A^{\mathtt{abc}}, \phi_{\mathtt{def}\,B}\} = \delta^{(\mathtt{a}}_{(\mathtt{d}}\delta^{\mathtt{b}}_{\mathtt{e}}\delta^{\mathtt{c})}_{\mathtt{f})}\delta_{AB}$$

Berman-Serre relations are again obeyed with

$$\hat{J}(\alpha) = X(\alpha)_{\mathtt{abc}\,\mathtt{def}}\,\phi^{\mathtt{abc}}\Gamma(\alpha)\phi^{\mathtt{def}}$$

and

$$\begin{aligned} X_{\mathtt{abc}}{}^{\mathtt{def}}(\alpha) \;=\; & -\frac{1}{3}\alpha_{\mathtt{a}}\alpha_{\mathtt{b}}\alpha_{\mathtt{c}}\alpha^{\mathtt{d}}\alpha^{\mathtt{e}}\alpha^{\mathtt{f}} + \frac{3}{2}\alpha_{(\alpha}\alpha_{\mathtt{b}}\delta^{(\mathtt{d}}_{\mathtt{c})}\alpha^{\mathtt{d}}\alpha^{\mathtt{e}}\alpha^{\mathtt{f})} - \frac{3}{2}\alpha_{(\mathtt{a}}\delta^{(\mathtt{d}}_{\mathtt{b}}\delta^{\mathtt{e}}_{\mathtt{c})}\alpha^{\mathtt{f})} \\ & + \frac{1}{4}\delta^{(\mathtt{d}}_{(\mathtt{a}}\delta^{\mathtt{e}}_{\mathtt{b}}\delta^{\mathtt{f})}_{\mathtt{c})} + \frac{1}{12}\big(2-\sqrt{3}\big)\alpha_{(\mathtt{a}}G_{\mathtt{bc})}G^{(\mathtt{de}}\alpha^{\mathtt{f})} \\ & \frac{1}{12}\big(-1+\sqrt{3}\big)\Big(\alpha_{\mathtt{a}}\alpha_{\mathtt{b}}\alpha_{\mathtt{c}}G^{(\mathtt{de}}\alpha^{\mathtt{f})} + \alpha_{(\mathtt{a}}G_{\mathtt{bc})}\alpha^{\mathtt{d}}\alpha^{\mathtt{e}}\alpha^{\mathtt{f}}\Big) \end{aligned}$$

As before, $\hat{J}(\alpha)$ provides a realisation *for all* real roots.

Conjecture: there exists an *infinite tower* of ever increasing finite-dimensional fermionic representations that capture more and more of $\mathrm{K}(\mathrm{E}_{10})$. [Kleinschmidt,HN(2013)]

Associated quotient groups $\mathcal{Q}_V = \mathrm{K}(\mathrm{E}_{10})/\mathcal{N}_V$ (with $\mathcal{N}_V \equiv$ "$\exp(\mathfrak{i}_V)$" = 'normal subgroup' associated with ideal $\mathfrak{i}_V$) can be viewed as *'generalized holonomy groups'*.

Major Challenges

- **Understanding $\mathrm{K(E_{10})}$, and thus $\mathrm{E_{10}}$, via an infinite tower of ever increasing unfaithful representations?**
  [cf. ongoing work with A. Kleinschmidt, R. Koehl and R. Lautenbacher]

- **Associated quotient groups $\mathcal{Q}_V = \mathrm{K(E_{10})}/\mathcal{N}_V$ would capture more and more of the *group* $\mathrm{K(E_{10})}$:**

$$\mathcal{Q}_{1/2} = SO(32) \, , \quad \mathcal{Q}_{3/2} = SO(32,288) \, ,$$
$$\mathcal{Q}_{5/2} = SO(288,1472) \, , \quad \mathcal{Q}_{7/2} = SO(1472,5568) \, , \, \cdots$$

  NB: $\mathcal{Q}_V$ *not* subgroups of $\mathrm{K(E_{10})}$, and non-compact!

- **$\Rightarrow$ $\mathrm{K(E_{10})}$ as some kind of projective limit?**

- **Probably needed to see how $\mathrm{K(E_{10})}$ 'unfolds' to give rise to emergent space-time fermions, and to see whether/how $\mathrm{K(E_{10})}$ is broken to SM symmetries.**

While it may take a long time to figure this out we can still search for *observable signatures* of this scheme.

Curious Gravitinos

[K.Meissner,HN: PRD100(2019)035001]

Under $SU(3)_c \times U(1)_{em}$ gravitinos transform as

$$\left(\mathbf{3}_c\,,\,\frac{1}{3}\right) \oplus \left(\bar{\mathbf{3}}_c\,,\,-\frac{1}{3}\right) \oplus \left(\mathbf{1}_c\,,\,\frac{2}{3}\right) \oplus \left(\mathbf{1}_c\,,\,-\frac{2}{3}\right)$$

Unusual features:

- **strong and electromagnetic interactions $\Rightarrow$**
- **would have been seen *unless* mass is very high, and cosmological abundance *extremely low***
- **would be stable against decay into SM matter because of peculiar quantum numbers $\Rightarrow$ can disappear only via mutual annihilation.**

[$\rightarrow$ very different from $N = 1$ MSSM gravitinos, which are uncharged under SM symmetries, and interact only weakly]

Not the usual Dark Matter Candidate

- **No SUSY: all gravitinos have masses** $\sim M_{\rm PL}$
- **Color triplet gravitinos should form (fractionally charged!) color singlet bound states with ordinary quarks** $\Rightarrow$ **all states stable despite large mass!**
- **DM mass density in solar system** $\sim 10^6$ **GeV/m**^{3} $\Rightarrow$ 10^{-13} **gravitinos/m**^{3} $\Rightarrow$ **flux** $\Phi \lesssim 0.003$ **m**$^{-2}$ **yr**$^{-1}$ **sr**$^{-1}$
- **Despite strong and electromagnetic interactions can easily pass through Earth because of large mass.**
- **Non-relativistic** $\Rightarrow$ **time of flight measurements?**
- **Idea: look for long ionized tracks in *ultrastable* material (rock, diamond,...?)** $\rightarrow$ ***need a 'paleo-detector'***

  [see e.g.:J.Bramante et al.,1803.08044[hep-ph];S.Baum et al., 1806.05991[astro-ph.CO]]

Explaining UHECRs?

[K.Meissner, HN: JCAP1909(2019)041]

New mechanism: **color triplet gravitinos could explain observed UHECR events via gravitino-antigravitino annihilation in the 'skin' of neutron stars, provided**

- **Gravitinos get absorbed into stars ...**
- **... and get 'compressed' in neutron stars so as to enable them to annihilate in appreciable rates**

New features:

- **could explain dominant appearance of ions (rather than protons) towards very highest energies**
- **with some 'reasonable' assumptions calculated event rates come close to the ones observed at Pierre Auger Observatory (in Argentina)**

$\Rightarrow$ **Hints of E_{10} and $K(E_{10})$ in the sky?**

Outlook

- E_{10} **and** $\mathrm{K}(\mathrm{E}_{10})$ **unify and generalize known duality symmetries of supergravity and string theory.**
- **Understanding** $\mathrm{K}(\mathrm{E}_{10})$ **fermions could greatly help towards understanding** E_{10} **(otherwise hopeless?)**
- **All results obtained so far indicate that** E_{10} **requires a setting beyond known concepts of space and time.**
- **However: explaining how this emergence works in detail remains** ***the*** **outstanding challenge!**
- **Intriguing links between** $\mathrm{K}(\mathrm{E}_{10})$ **and SM fermions:** $\rightarrow$ **can** E_{10} **and** $\mathrm{K}(\mathrm{E}_{10})$ ***supersede supersymmetry*** **as a guiding principle towards unification?**
- **Ultimate hope: no multiverse, but an actual explanation why low energy world is the way it is...**

Points de Rencontre

- B. Julia and HN, ***"Null-Killing vector dimensional reduction and Galilean geometrodynamics"***, Nucl. Phys. B439 (1995) 291

- B. Julia and HN, ***"Conformal internal symmetry of 2d sigma models coupled to gravity and a dilaton"***, Nucl. Phys. B482 (1996) 431

- T. Damour, M. Henneaux, B. Julia and HN, ***"Hyperbolic Kac-Moody algebras and chaos in Kaluza-Klein models"***, Phys. Lett. B509 (2001) 323

Joyeux Emeritat et Joyeux Noël !