On Broken Supersymmetry and Vacuum Stability in Supergravity and String Theory

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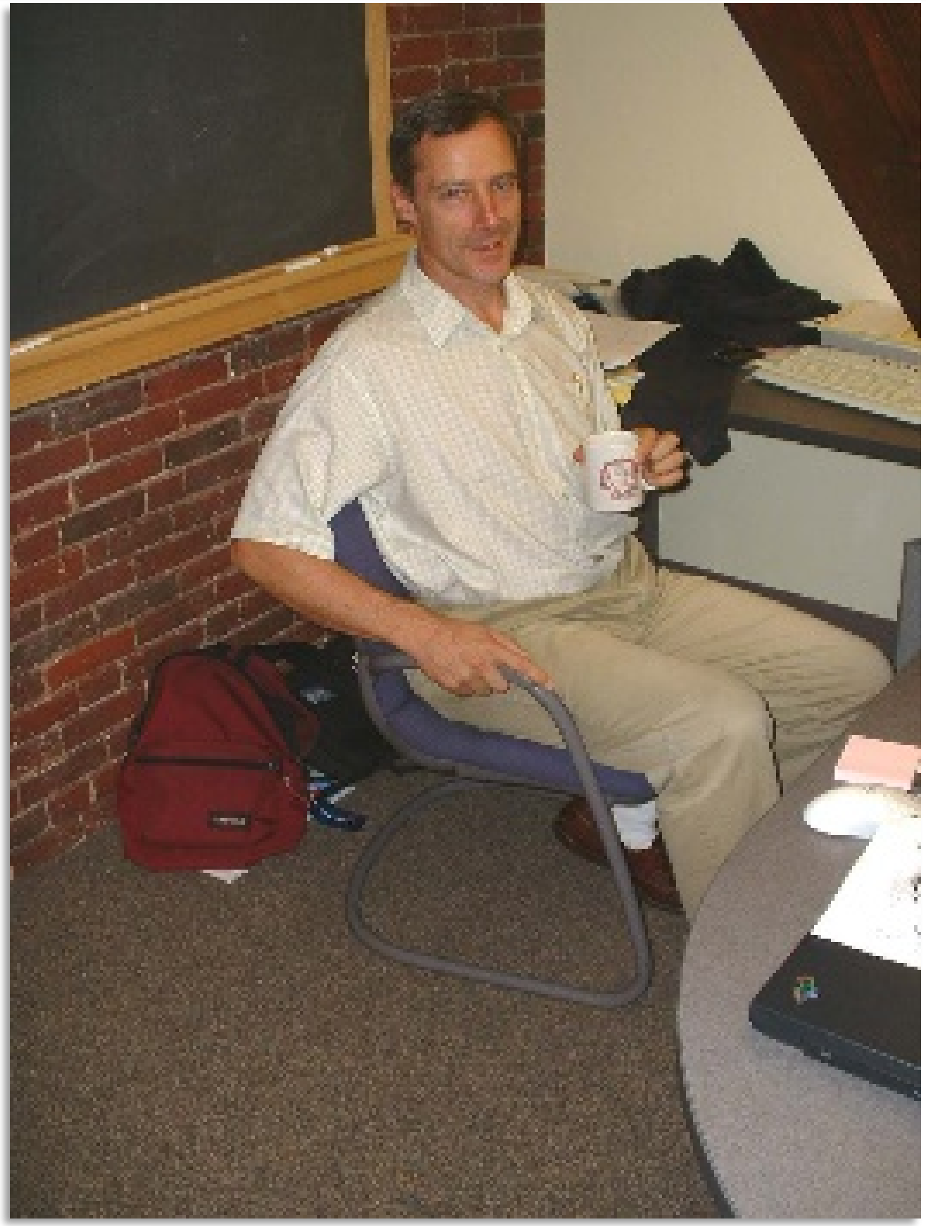
Based on: 1811.11448, 1711.11494, 1612.08566[hep-th]
and refs therein
AS and J. Mourad, to appear

Julia Fest ENS - Paris, December 16-17, 2019

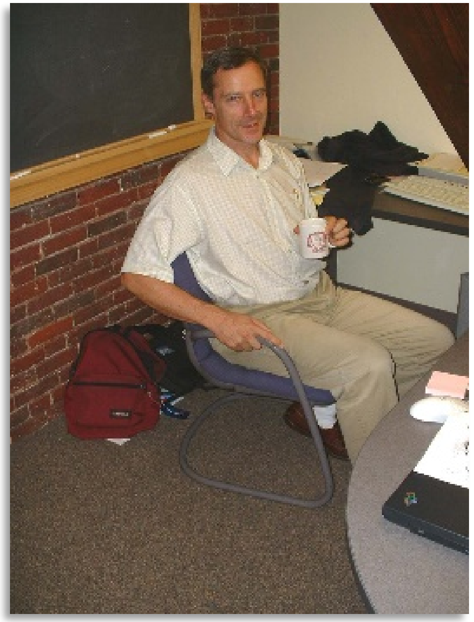
Istituto Nazionale di Fisica Nucleare

Alexander von Humboldt Stiftung/Foundation

We are here to celebrate our friend Bernard Julia



ALSO: a Trio and a Masterpiece (Eugene Cremmer, Bernard Julia and Joel Scherk)


$$
\begin{aligned}
\mathcal{S} \;=\; & \frac{1}{2\,k_{11}^2} \int d^{11}x\, e \Big[e_A^M\, e_B^N\, R_{MN}{}^{AB}(\omega) \\
& -\; i\,\overline{\psi}_M\, \gamma^{MNP}\, D_N \left(\frac{\omega+\hat{\omega}}{2}\right) \psi_P \;-\; \frac{1}{24}\, F_{ABCD}\, F^{ABCD} \\
& -\; \frac{i\sqrt{2}}{192} \left(\overline{\psi}_M\, \gamma^{MNABCD}\, \psi_N \;+\; 12\, \overline{\psi}^A\, \gamma^{CD}\, \psi^B\right) \left(F_{ABCD} \;+\; \widehat{F}_{ABCD}\right) \\
& -\; \frac{2\sqrt{2}}{(144)^2}\, \epsilon^{A_1\ldots A_4 B_1 \ldots B_4 MNP}\; F^{A_1\ldots A_4}\; F^{B_1\ldots B_4}\, A_{MNP} \Big]
\end{aligned}
$$

$$\delta\, e_M^A \;=\; \frac{i}{2}\, \overline{\epsilon}\, \gamma^A\, \psi_M \,,$$

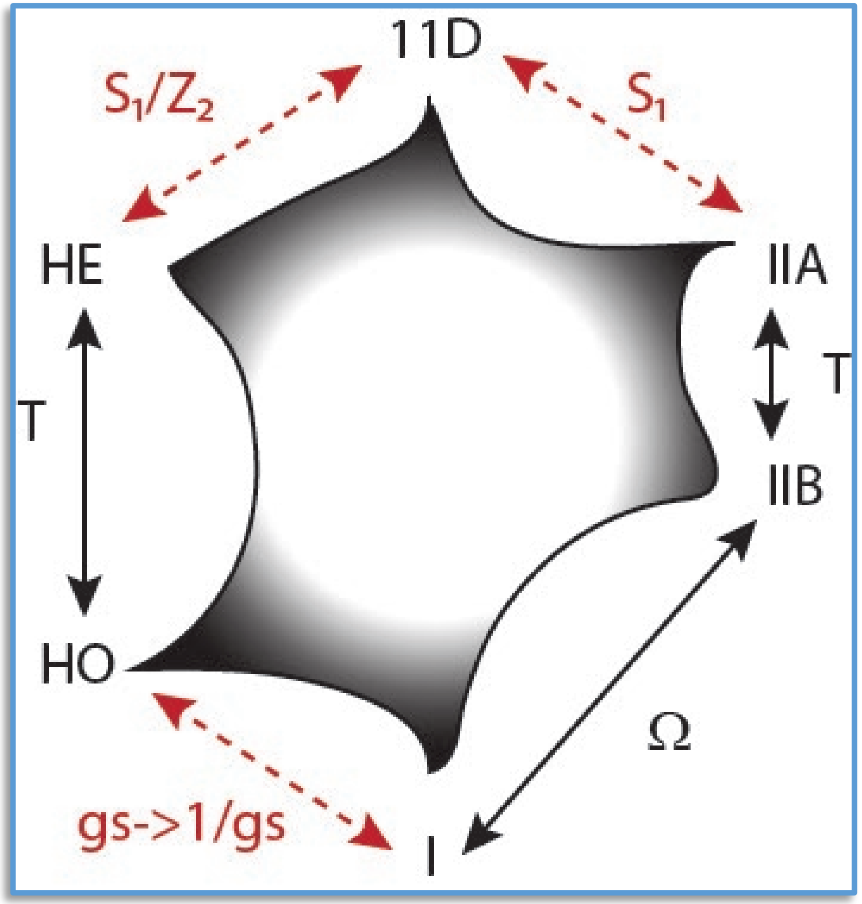
$$\delta\, \psi_\mu \;=\; D_M \epsilon \;+\; \frac{\sqrt{2}}{288} \left(\gamma^{ABCD}{}_M \;-\; 8\, \delta_M^A\, \gamma^{BCD}\right) F_{ABCD}\, \epsilon \,,$$

$$\delta\, A_{MNP} = \;-\; \frac{3\sqrt{2}\, i}{4}\; \overline{\epsilon}\, \gamma_{[MN}\, \psi_{P]}$$

The Problem

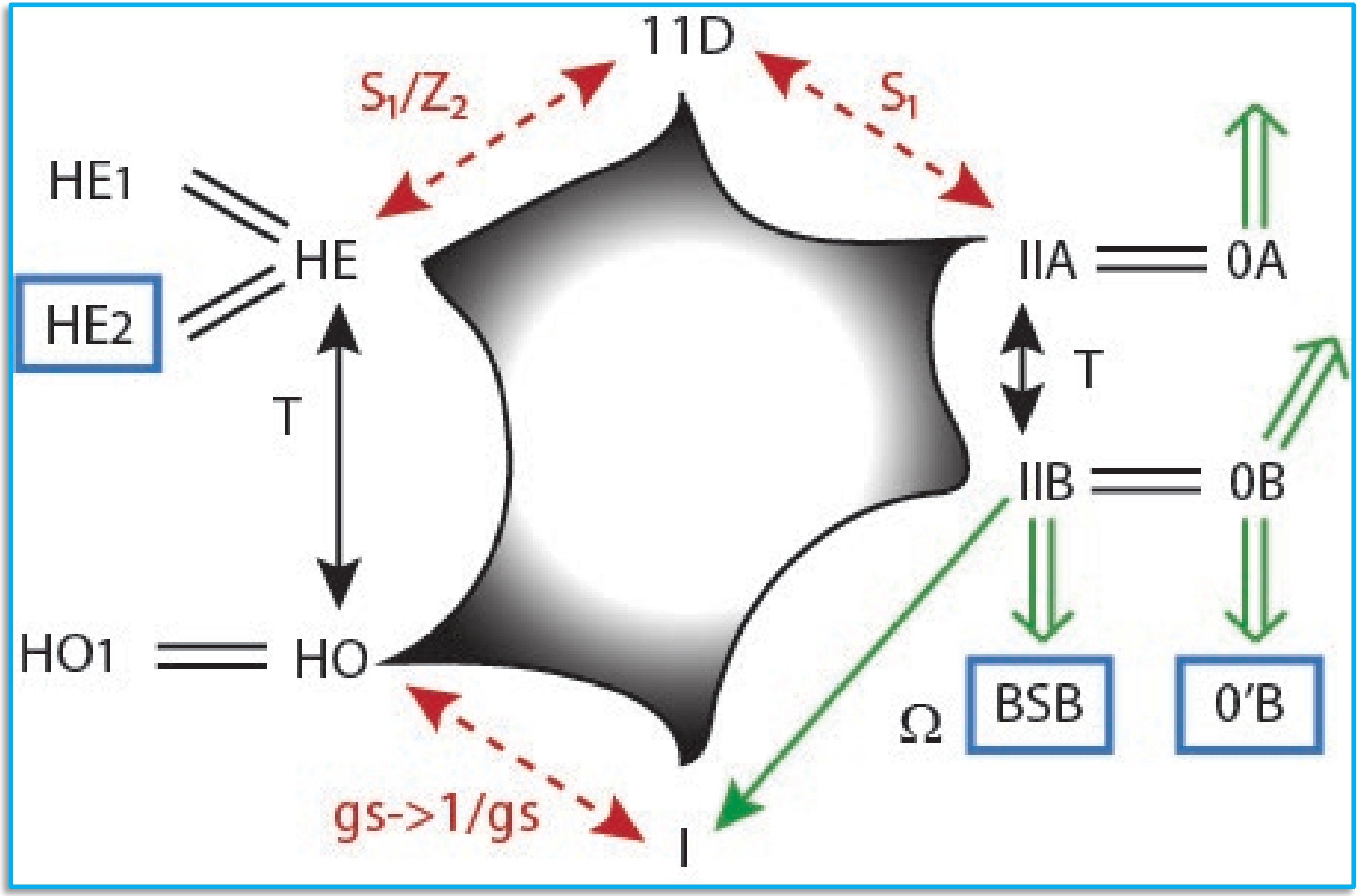
The (SUSY) 10D-11D Zoo

- 11D: highest point of (SUSY) String Theory
- Exhibits dramatically our limits
- Solid arrows → perturbative
- 10&11D supergravity → Dashed arrows
- SUSY: stabilizes these 10D Minkowski vacua



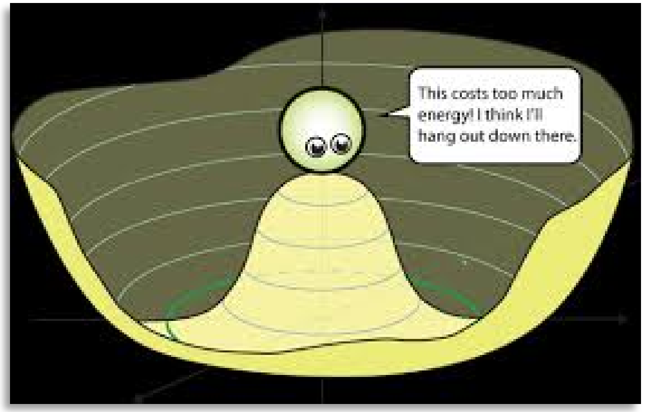
BROKEN SUSY ?

The 10D-11D Zoo



Vacuum Configurations & String Theory

- NO SUSY → TACHYONS !



- NO – TACHYONS ? MAJOR RESULT → G. S. O. & SUSY-SUGRA

(Gliozzi, Scherk, Olive, 1977)

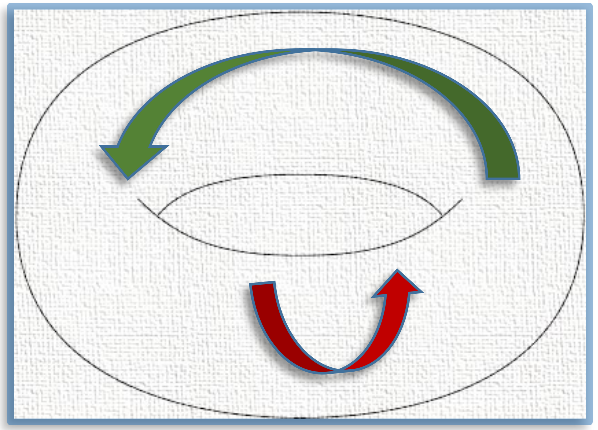
- MORE GENERALLY NO TACHYONS → YES !

❖ 3 D=10 S~~U~~SY STRINGS:

- SO(16)xSO(16) (HETEROTIC) *(Dixon, Harvey, 1986)* *(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1986)*
- U(32) O'B (ORIENTIFOLD) *(AS, 1995)*
- Usp(32) (ORIENTIFOLD) → "Brane SUSY Breaking" (HIDDEN SUSY)

(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

10D (Closed) Superstrings



❖ Building principles of (closed) string spectra & vacuum energy:

- spin-statistics *(GSO projections)*
- modular invariance

(Gliozzi, Scherk, Olive, 1977)

- IIA, IIB:

$$\mathcal{T}_{IIA} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{(V_8 - S_8)(\bar{V}_8 - \bar{C}_8)}{(Im\tau)^4 \, \eta^8 \, \bar{\eta}^8} \qquad \mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|V_8 - S_8|^2}{(Im\tau)^4 \, \eta^8 \, \bar{\eta}^8}$$

$$IIA: \ (e_\mu^a, B_{\mu\nu}, \phi, \psi_{\mu\, L+R}, \psi_{L+R}, A_\mu, C_{\mu\nu\rho})$$
$$IIB: \ (e_\mu^a, B_{\mu\nu}^{1,2}, \phi^{1,2}, \psi_{\mu\, L}^{1,2}, \psi_R^{1,2}, D_{\mu\nu\rho\sigma}^+)$$

- HE, HO:

$$\mathcal{T}_{HE} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2} \frac{(V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})^2}{Im\tau^4 \, \eta^8 \, \bar{\eta}^8} \qquad \mathcal{T}_{HO} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2} \frac{(V_8 - S_8)(\bar{O}_{32} + \bar{S}_{32})}{Im\tau^4 \, \eta^8 \, \bar{\eta}^8}$$

$$HE: \ (e_\mu^a, B_{\mu\nu}, \phi, \psi_{\mu\, L}, \psi_R) \oplus (E_8 \times E_8 \text{ super YM})$$
$$HO: \ (e_\mu^a, B_{\mu\nu}, \phi, \psi_{\mu\, L}, \psi_R) \oplus (SO(32) \text{ super YM})$$

SUSY: $V_8 = S_8 = C_8$

- 0A, 0B:

$$\mathcal{T}_{0A} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|O_8|^2 + |V_8|^2 + S_8\,\bar{C}_8 + C_8\,\bar{S}_8}{(Im\tau)^4 \, \eta^8 \, \bar{\eta}^8} \qquad \mathcal{T}_{0B} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4 \, \eta^8 \, \bar{\eta}^8}$$

$$0A: \ (e_\mu^a, B_{\mu\nu}, \phi, T, A_\mu^{1,2}, C_{\mu\nu\rho}^{1,2})$$
$$0B: \ (e_\mu^a, B_{\mu\nu}^{1,2,3}, \phi^{1,2,3}, T, D_{\mu\nu\rho\sigma})$$

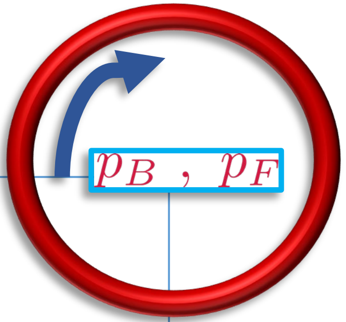
❖ H_{16x16}:

$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2} \frac{1}{Im\tau^4 \, \eta^8 \, \bar{\eta}^8} \left[O_8(\bar{V}_{16}\,\bar{C}_{16} + \bar{C}_{16}\,\bar{V}_{16}) + V_8(\bar{O}_{16}\,\bar{O}_{16} + \bar{S}_{16}\,\bar{S}_{16}) - S_8(\bar{O}_{16}\,\bar{S}_{16} + \bar{S}_{16}\,\bar{O}_{16}) - C_8(\bar{V}_{16}\,\bar{V}_{16} + \bar{C}_{16}\,\bar{C}_{16}) \right]$$

$$(e_\mu^a, B_{\mu\nu}, \phi) \oplus A_\mu^{(120,1)\oplus(1,120)} \oplus \psi_L^{(128,1)\oplus(1,128)} \oplus \psi_R^{(16,16)}$$

(Dixon, Harvey; Seiberg, Witten, 1986)
(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1986)

SUSY Breaking (in String Theory)



- (CIRCLE) SCHERK-SCHWARZ: MODIFY KALUZA-KLEIN FOR FERMIONS

$$p_B = \frac{m}{R} \quad [\psi_B(x + 2\,\pi\,R) = \psi_B(x)] \qquad p_F = \frac{m+\frac{1}{2}}{R} \quad [\psi_F(x + 2\,\pi\,R) = -\,\psi_F(x)]$$

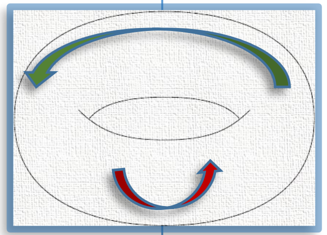
(Scherk Schwarz, 1979)

- [N=8 SUGRA & DIVERGENCE CANCELATIONS] (Cremmer, Scherk, Schwarz, 1979; Segin, van Niuwenhuizen, 1982)

- BUT: MORE SUBTLE & COMPLICATED IN STRING THEORY

(Rohm, 1984; Ferrara, Kounnas, Porrati, Zwirner, 1989)

$$\mathcal{T}_{IIB} \to \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\, \frac{1}{(Im\tau)^{\frac{7}{2}}\, \eta^8\, \bar\eta^8} \times \Big[(V_8\,\bar V_8 + S_8\,\bar S_8) \sum_{m,n} \frac{1+(-1)^m}{2}\, \Lambda_{m,n} - (V_8\,\bar S_8 + S_8\,\bar V_8) \sum_{m,n} \frac{1-(-1)^m}{2}\, \Lambda_{m,n} + (O_8\,\bar O_8 + C_8\,\bar C_8) \sum_{m,n} \frac{1+(-1)^m}{2}\, \Lambda_{m,n+\frac{1}{2}} - (O_8\,\bar C_8 + C_8\,\bar O_8) \sum_{m,n} \frac{1-(-1)^m}{2}\, \Lambda_{m,n+\frac{1}{2}} \Big]$$

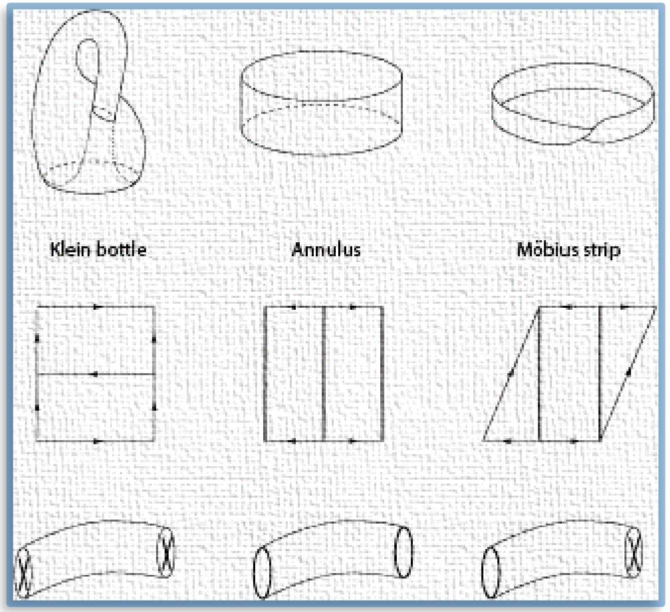


- [MODULAR INVARIANCE → twisted sector with TACHYONS if $|M_B - M_F| \gtrsim \frac{1}{\sqrt{\alpha'}}$!!]

10D Tachyon-Free Orientifolds

$(e_\mu^a, B_{\mu\nu}, D^+_{\mu\nu\rho\sigma}, \phi, \phi') \oplus (A_{\mu\, a}{}^{b}, \lambda^{[ab]}, \lambda_{[ab]})\ , \quad \mathrm{U}(32)$ gauge group

(AS, 1995)



$$\mathcal{T}_{0'B} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\, \frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\, \eta^8\, \bar{\eta}^8}[\tau,\bar{\tau}] \qquad \mathcal{K}_{0'B} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{-O_8 - V_8 + S_8 - C_8}{(\tau_2)^4\, \eta^8}[2i\tau_2]$$

$$\mathcal{A}_{0'B} = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\mathcal{N}\bar{\mathcal{N}}\, V_8 - \frac{\mathcal{N}^2+\bar{\mathcal{N}}^2}{2}\, C_8}{(\tau_2)^4\, \eta^8}[i\tau_2/2] \qquad \mathcal{M}_{0'B} = \frac{\mathcal{N}+\bar{\mathcal{N}}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\hat{C}_8}{(\tau_2)^4\, \hat{\eta}^8}[i\tau_2/2+1/2]$$

$$\mathcal{N} = \bar{\mathcal{N}} = 32$$

$(e_\mu^a, B_{\mu\nu}, \phi, \psi_\mu, \psi) \oplus (A_\mu^{(ab)}, \lambda^{[ab]})\ , \quad \mathrm{USp}(32)$ gauge group

(Sugimoto, 1999)

$$\mathcal{T}_{Usp(32)} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\, \frac{|V_8 - S_8|^2}{(Im\tau)^4\, \eta^8\, \bar{\eta}^8}[\tau,\bar{\tau}] \qquad \mathcal{K}_{USp(32)} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{V_8 - S_8}{(\tau_2)^4\, \eta^8}[2i\tau_2]$$

$$\mathcal{A}_{Usp(32)} = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{V_8 - S_8}{(\tau_2)^4\, \eta^8}[i\tau_2/2] \qquad \mathcal{M}_{USp(32)} = -\frac{\mathcal{N}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{(\text{“}-\text{''}\, \hat{V}_8 - \hat{S}_8)}{(\tau_2)^4\, \hat{\eta}^8}[i\tau_2/2+1/2]$$

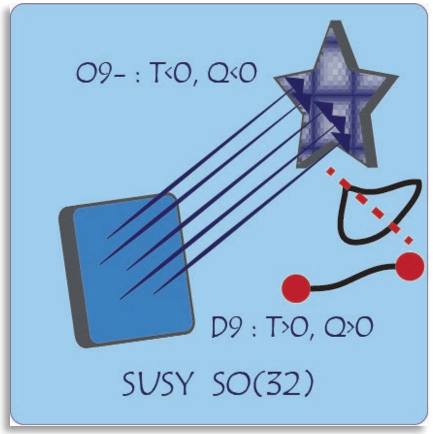
$$\mathcal{N} = 32$$

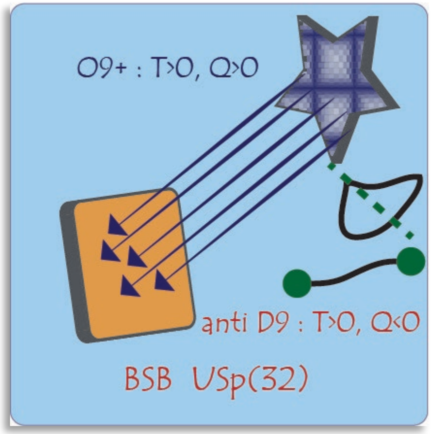
Only sign flip w.r.t. type-I SUSY SO(32)!

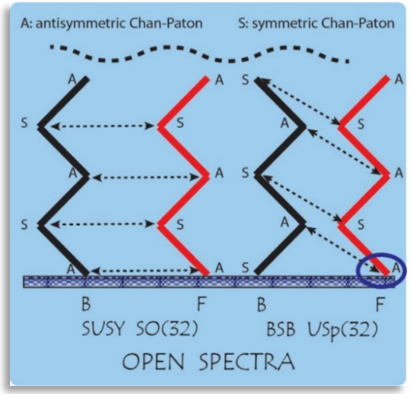
Brane SUSY Breaking in String Theory

- (Non-linear) SUSY
- NO TACHYONS

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)







$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{-\phi}+\ldots\right\}$$

- SO(32): N(N-1)/2 vectors and spinors
- Usp(32): N(N+1)/2 vectors, N(N-1)/2 spinors ⊃ GOLDSTINO

VACUUM ENERGY → POTENTIAL

(Dudas, Mourad, 2000)

[Constrained Superfields in D=4]

- Consider an O(N) invariant scalar model:
  - For $\lambda \to \infty$: a σ-model describing the dynamics in the valley of minima

$$S \;=\; \int d^D x \left[-\,\frac{1}{2}\, \partial_\mu \phi^T\, \partial^\mu \phi \;-\; \lambda \left(\phi^T\, \phi \;-\; \rho^2\right)^2 \right]$$

$$S \;=\; -\int d^D x\, \frac{1}{2}\, \partial_\mu \phi^T\, \partial^\mu \phi\,, \qquad \phi^T\, \phi \;=\; \rho^2$$

- In Wess-Zumino multiplet of 4D SUSY, when the scalar becomes very massive
  - A fermion with broken SUSY.
    Volkov-Akulov (1973) model (up to field redefinitions)

(Casalbuoni, De Curtis, Dominici, Feruglio, Gatto, 1989)
(Komargodski, Seiberg, 2009)

$$S \;=\; \int d^4 x\; d^2\,\theta\, d^2\,\overline{\theta}\left[\,\overline{\Phi}\,\Phi \;+\; \lambda\,\left(\overline{\Phi}\,\Phi\right)^2\,\right] \;+\; \int d^4 x\; d^2\,\theta\, f\,\Phi \;+\; \text{h.c.}$$

$$\lambda \;\to\; \infty \;\;: \qquad \Phi \;=\; \frac{\psi^\alpha\,\psi_\alpha}{2\,F} \;+\theta^\alpha\,\psi_\alpha \;+\frac{\theta^2}{2}\;F\;, \qquad \Phi^2 \;=\; 0 \quad [F \neq 0]$$

"Vacuum" Solutions with Broken SUSY

Cosmological Potentials

- What potentials lead to slow-roll, and where ?

$$ds^2 = -\,dt^2 + e^{2A(t)}\,d\mathbf{x}\cdot d\mathbf{x} \quad \Longrightarrow \quad \ddot{\phi} + 3\,\dot{\phi}\sqrt{\frac{1}{3}\,\dot{\phi}^2 + \frac{2}{3}\,V(\phi)} + V' = 0$$

Driving force from V′ *vs* friction from V

- If V does not vanish : convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\,dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\,d\mathbf{x}\cdot d\mathbf{x}$$

$$V\,e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{(d-1)}{2(d-2)}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \frac{V_\varphi}{2\,V}\,(1 + \dot{\varphi}^2) = 0$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2\,V} = \frac{n}{2\,\varphi}$$

❖ Quadratic potential? Far away from origin *(Linde, 1983)*

❖ Exponential potential? YES or NO

$$V(\varphi) = V_0\,e^{2\gamma\varphi} \longrightarrow \frac{V'}{2\,V} = \gamma$$

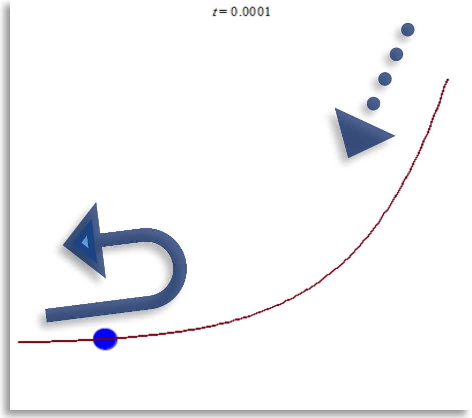
The Climbing Scalar

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004)
(Dudas, Kitazawa, AS, 2010)
(Dudas, Condeescu, 2013)

- $\gamma < 1$? Both signs of speed

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \gamma\left(1+\dot{\varphi}^2\right) = 0$$

a. "Climbing" solution (φ climbs, then descends → e^{φ} BOUNDED)
b. "Descending" solution (φ only descends)



Limiting τ- speed (LM attractor): $v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$

(Lucchin and Matarrese, 1985)

$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond !

CLIMBING : BSB (Usp(32)) and U(32) HAVE PRECISELY $\gamma = 1$!
[ALL 3 TACHYON-FREE MODELS → exponential potentials with $\gamma \geq 1$] !

- $\gamma = 1$:

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$

$$\mathcal{A}(\tau) = \mathcal{A}_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$

Critical Exponentials and BSB

(Dudas, Kitazawa, AS, 2010)
(AS, 2013)
(Fré, AS, Sorin, 2013)

$$\mathcal{S} = \frac{1}{2\,k_N^2} \int d^{10}x\,\sqrt{-\det g}\,\left[R + \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - T\,e^{\frac{3}{2}\,\phi} + \dots\right] \longrightarrow \gamma = 1\,(\text{for } \varphi)$$

- D< 10 : two combinations of ϕ and "breathing mode" $\sigma \rightarrow (\Phi_s, \Phi_t)$
- Φ_t yields a "critical" φ potential ($\gamma = 1$) if Φ_s is stabilized

$$S_d = \frac{1}{2\kappa_d^2} \int d^dx\sqrt{-g}\left[R + \frac{1}{2}\,(\partial\Phi_s)^2 + \frac{1}{2}\,(\partial\Phi_t)^2 - T_9\,e^{\sqrt{\frac{2(d-1)}{d-2}}\,\Phi_t} + \dots\right]$$

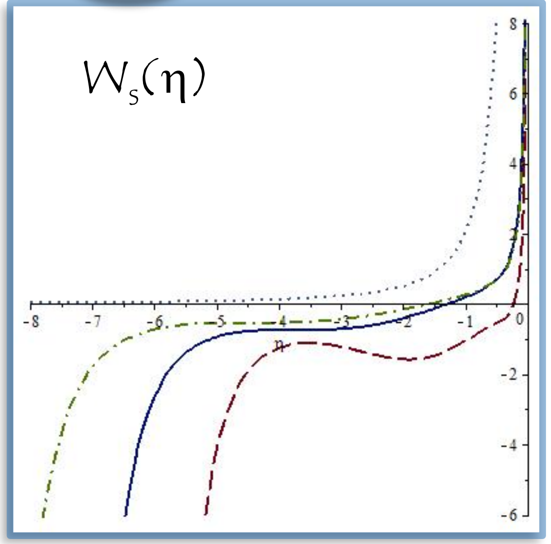
- If Φ_s is stabilized: a p-brane that couples via $(g_s)^{-\alpha}$
  [the D9-brane we met before had p=9, α=1]

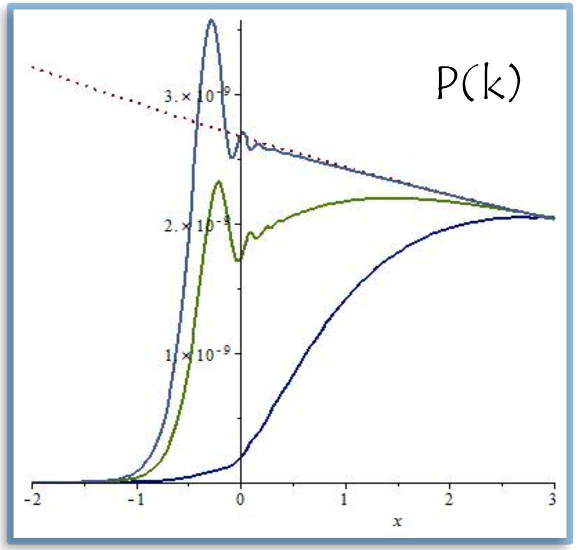
$$\gamma = \frac{1}{12}\,(p + 9 - 6\,\alpha) \quad \Longrightarrow \quad V = V_0\left(e^{2\varphi} + e^{2\gamma\varphi}\right) \quad (\gamma < 1)$$

Fast-Roll and Mukhanov-Sasaki Equation

(Dudas, Kitazawa, Patil, AS, 2013)
(Kitazawa, AS, 2014)

- MS equation : $$\left(\frac{d^2}{d\eta^2} + k^2 - W_s(\eta)\right) v_k(\eta) = 0$$
- Limiting W_s : $$W_s \underset{\eta\to-\eta_0}{\sim} -\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2} \qquad W_s \underset{\eta\to-0}{\sim} \frac{\nu^2-\frac{1}{4}}{\eta^2} \qquad \left(\nu = \frac{3}{2}\,\frac{1-\gamma^2}{1-3\,\gamma^2}\right)$$
- Power : $$P(k) = \frac{k^3}{2\,\pi^2}\left|\frac{v_k(-\epsilon)}{z(-\epsilon)}\right|^2$$

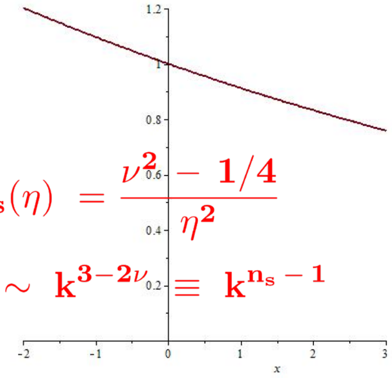




❖ Pre – inflationary fast roll : $P(k) \sim k^3$

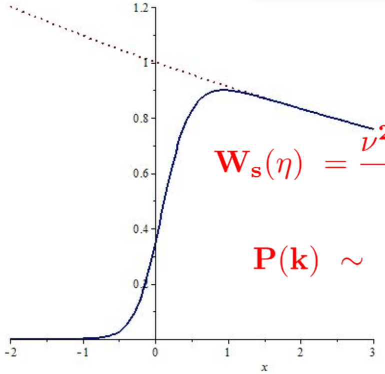
WKB : $$v_k(-\epsilon) \sim \frac{1}{\sqrt[4]{|W_s(-\epsilon) - k^2|}}\,\exp\left(\int_{-\eta^\star}^{-\epsilon}\sqrt{|W_s(y) - k^2|}\,dy\right)$$

(Chibisov, Mukhanov, 1981)



$$\mathbf{W_s}(\eta) = \frac{\nu^2 - 1/4}{\eta^2}$$

$$\mathbf{P(k)} \sim \mathbf{k^{3-2\nu}} \equiv \mathbf{k^{n_s-1}}$$



$$\mathbf{W_s}(\eta) = \frac{\nu^2 - 1/4}{\eta^2} - \Delta^2$$

$$\mathbf{P(k)} \sim \frac{\mathbf{k^3}}{[\mathbf{k^2} + \Delta^2]^\nu}$$

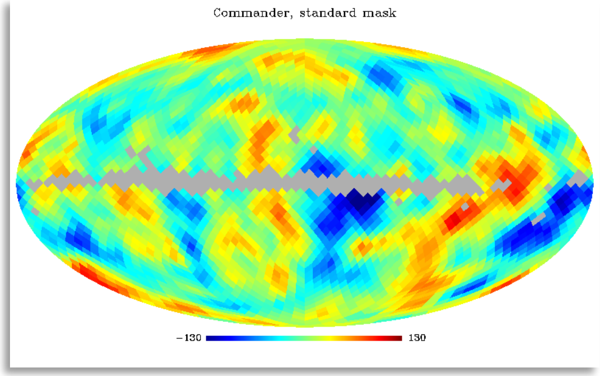
Pre-Inflationary Relics in the CMB?

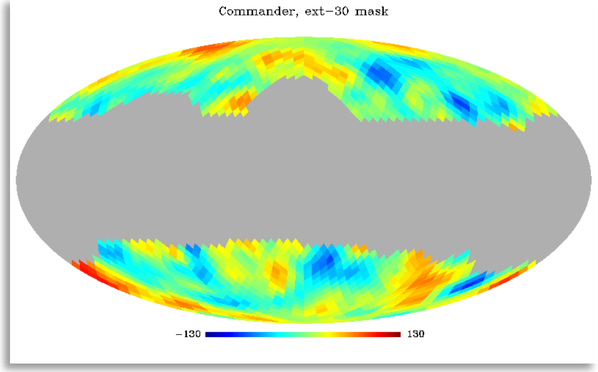
(Gruppuso, Mandolesi, Natoli, Kitazawa, AS, 2015)

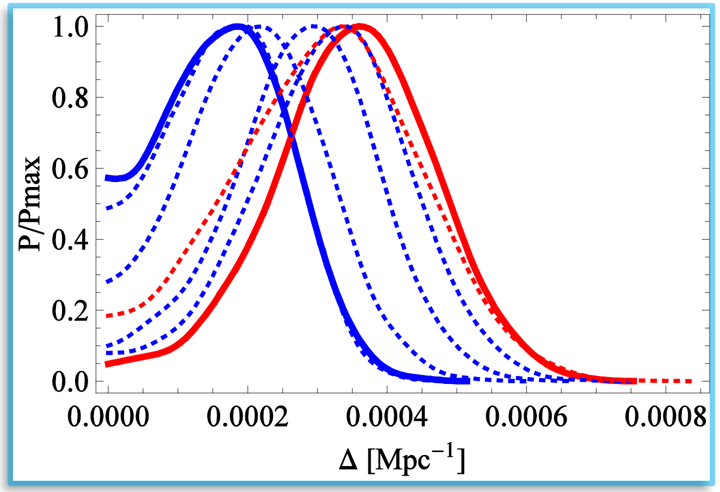
- Extend ΛCDM to allow for low-ℓ suppression:

$$\mathcal{P}(k) \;=\; A\;(k/k_0)^{n_s-1} \;\rightarrow\; \frac{A\;(k/k_0)^3}{\left[(k/k_0)^2+(\Delta/k_0)^2\right]^{\nu}}$$

- NO effects on standard ΛCDM parameters (6+16 nuisance)
- A new scale Δ. Preferred value? Depends on GALACTIC MASK







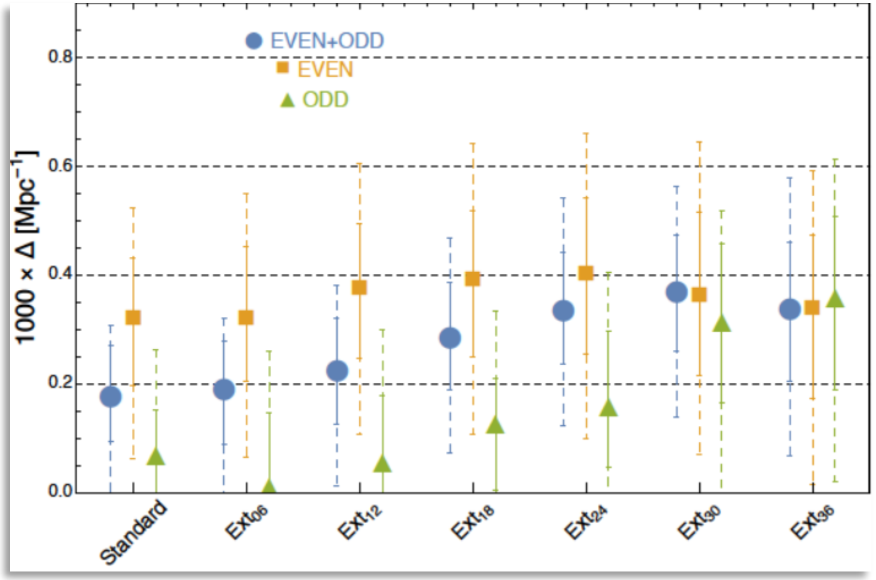
$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

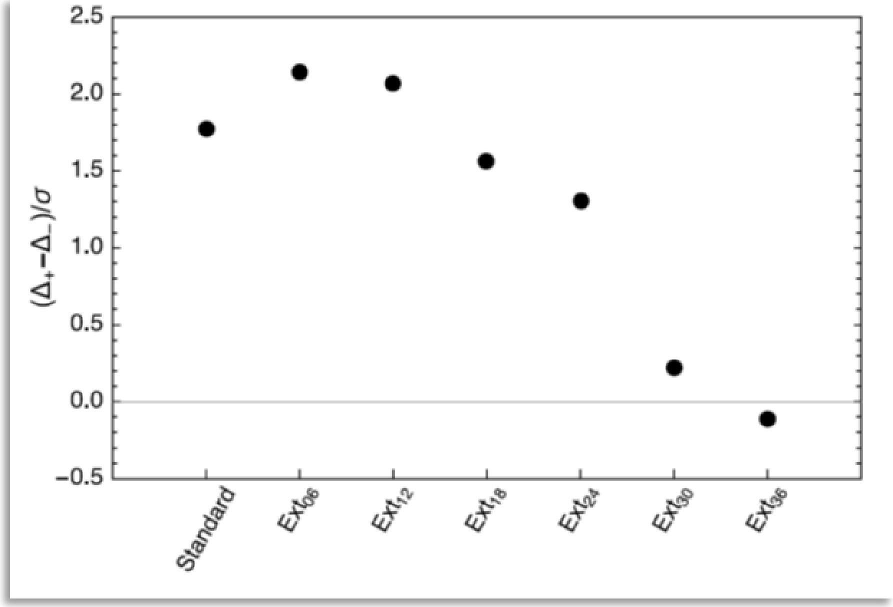
RED : + 30−degree extended mask

$> 99\%$ confidence level

$$\Delta^{Infl} \;\sim\; 2.4 \times 10^{12}\; e^{N-60}\;\mathrm{GeV} \;\sim\; 10^{12} - 10^{14}\mathrm{GeV} \;\; \text{for} \;\; \mathrm{N} \;\sim\; 60 - 65$$

(Even & Odd) Detections of Δ

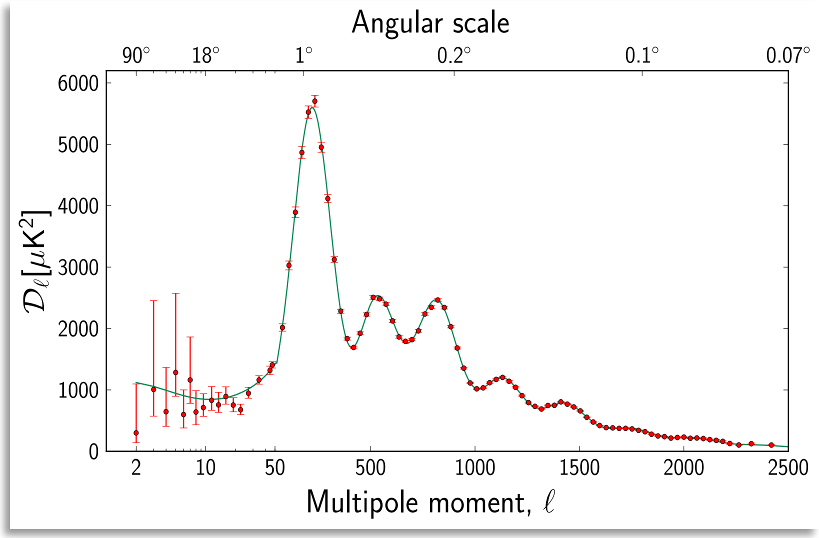




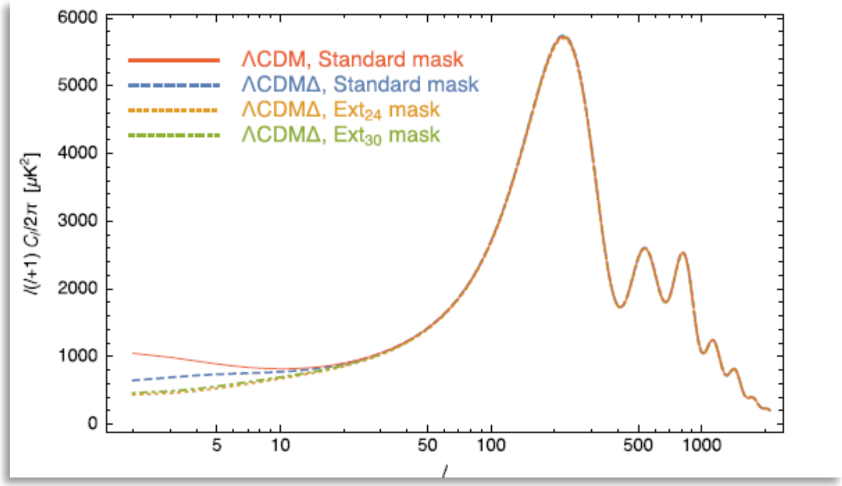
(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)

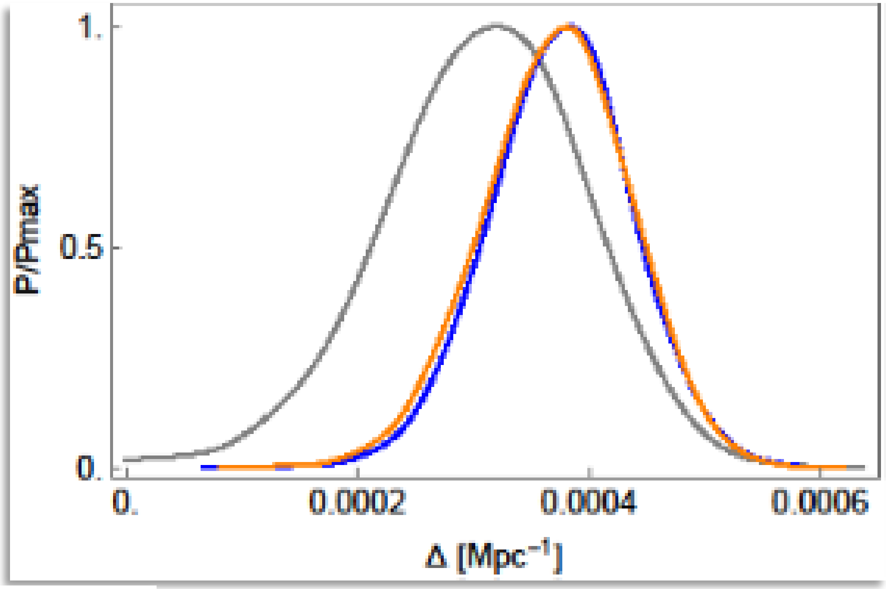
Case	Label	dataset	Detection level (%)	Detection level (σ)
a	Standard	full	93.26	1.83
a	Standard	even	98.59	2.46
a	Standard	odd	52.52	0.72
b	Ext_{06}	full	92.30	1.77
b	Ext_{06}	even	98.65	2.47
b	Ext_{06}	odd	41.03	0.54
c	Ext_{12}	full	96.41	2.10
c	Ext_{12}	even	99.39	2.74
c	Ext_{12}	odd	18.93	0.24
d	Ext_{18}	full	99.15	2.63
d	Ext_{18}	even	99.23	2.67
d	Ext_{18}	odd	69.80	1.03
e	Ext_{24}	full	99.32	2.71
e	Ext_{24}	even	99.05	2.59
e	Ext_{24}	odd	81.57	1.33
f	Ext_{30}	full	99.84	3.16
f	Ext_{30}	even	98.47	2.43
f	Ext_{30}	odd	94.37	1.91
g	Ext_{36}	full	98.60	2.46
g	Ext_{36}	even	96.27	2.08
g	Ext_{36}	odd	96.60	2.12

Prospects



(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)





- Δ does not affect standard ΛCMB parameters

WHAT NEXT? POLARIZATION

- Cosmic-variance limited E-mode could allow a 5-6 σ detection of Δ (or could rule it out)
- Enhanced tensor (B) mode around the scale Δ (+large-scale suppression)
- [Enhanced non-gaussianity ? ~ (n_s-1)]

9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{-R - \frac{1}{2}\,(\partial\phi)^2 - \frac{1}{12}\,e^{\frac{3-p}{2}\,\phi}\,\mathcal{H}_3^2 - T\,e^{\frac{3}{2}\,\phi} + \,\ldots\right\}$$

9D solution → (INTERVAL) COMPACTIFICATION

Singularities at the ends (for the space-time geometry and for the string coupling)

$$\phi = \frac{3}{4}\,\alpha_O y^2 + \frac{1}{3}\,\ln\left|\alpha_O y^2\right| + \Phi_0 \qquad (\alpha_O \sim T)$$

$$ds_O^2 = \left|\alpha_O\,y^2\right|^{\frac{1}{18}} e^{-\alpha_O\frac{y^2}{8}}\,\eta_{\mu\nu}dx^\mu dx^\nu + e^{-\frac{3}{2}\,\Phi_0}\left|\alpha_O y^2\right|^{-\frac{1}{2}} e^{-\frac{9}{8}\,\alpha_O y^2}\,dy^2$$

[Applies to both Usp(32) and U(32), similar but more complicated results for heterotic SO(16) x SO(16)]

a) string loop corrections: grow out of control for y→ ∞;

b) curvature corrections: similar problem near y=0.

c) BUT: (BRANE) SUSY BREAKING induces a COMPACTIFICATION → FINITE length ~ $\frac{1}{\sqrt{T}}$

d) FINITE 9D Planck mass and gauge coupling

Orientifold Flux Vacua with BSB

(Mourad, AS, 2016)

- In this setting the field equations from

$$\mathcal{S} \;=\; \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{-R-\frac{1}{2}\,(\partial\phi)^2-\frac{1}{12}\,e^{\frac{3-p}{2}\,\phi}\,\mathcal{H}_3^2\;-\;T\,e^{\frac{3}{2}\,\phi}\;+\;\ldots\right\}$$

reduce to

$$(\star):\;T\,e^{\gamma_E\,\phi} \;=\; -\,\frac{\beta_E^{(p)}\,h^2}{\gamma_E}\;e^{-\,2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{-\,2\,C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-\,2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\,e^{-\,2\,A}\;+\;\frac{h^2}{16(p\,+\,1)}\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\,e^{-\,2\,(8-p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

- (*): Dilaton Eq: constraint from positivity of *l.h.s.* (β_E < 0 for orientifolds & T>0, NEED H_3 fluxes)
- Third Eq: determines for k=0 A ~r, and thus AdS in Poincaré coordinates (or in other slicings for k = ±1)

❖ WIDE REGIONS where the two couplings $\alpha' R$ and $g_s \;=\; e^{\phi}$ are SMALL

Stability ?

Breitenlohner – Freedman Bounds

- Poincaré coordinates:

$$ds^2 = R_{AdS}^2 \frac{dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu}{z^2}$$

- E.g: Klein-Gordon eq.:

$$g^{MN} \nabla_M \nabla_N \Phi - (M R_{AdS})^2 \Phi = 0$$
$$\phi(x,z) = e^{ik\cdot x} z^{\frac{d}{2}-1} \Psi(z)$$

- Schrödinger-like:

$$\left[- \frac{d^2}{dz^2} + \frac{4 (M R_{AdS})^2 + d(d-2)}{4 z^2} \right] \Psi = - k^2 \Psi$$
$$\mathcal{A} = - \frac{d}{dz} + \frac{a}{z}, \quad \mathcal{A}^\dagger = \frac{d}{dz} + \frac{a}{z}$$
$$\mathcal{A}^\dagger \mathcal{A} \Psi = -k^2 \Psi$$

- BF Bound:

$$\left(a - \frac{1}{2}\right)^2 = (M R_{AdS})^2 + \frac{(d-1)^2}{4} \geq 0$$

$AdS_3 \times S^7$ (& $AdS_7 \times S^3$) Vacua

(Basile, Mourad, AS, 2018)

- **Basic Equation:** $$g_{\mu\nu}\, A \,+\, \nabla_\mu \nabla_\nu\, B \,=\, 0 \;\longrightarrow\; A = 0\,, \quad B = 0$$
- **Scalar Perturbations:** $$\begin{aligned} b_{\mu\nu} &= \sqrt{-\lambda}\, \epsilon_{\mu\nu\rho}\, \nabla^\rho\, B \\ h_{\mu\nu} &= \lambda_{\mu\nu}\, A\,, \\ h_{\mu i} &= R^2\, \nabla_\mu\, \nabla_i\, D\,, \\ h_{ij} &= \gamma_{ij}\, C\,, \end{aligned}$$
- **Breitenlohner-Freedman Bound:** $$M^2 \,\geq\, -\, \frac{(d-1)^2}{4\, R_{AdS}^2}$$ (Breitenlohner, Freedman, 1982)
- **OK for $\ell=0$:** $$\begin{aligned} &\Box\, B \,+\, 4\left(\frac{1}{R_{AdS}^2} + \frac{3}{R^2}\right)(\varphi \,+\, 14\, C) \,=\, 0 \\ &\Box\, \varphi \,-\, V_0''\, \varphi \,-\, \left(\frac{1}{R_{AdS}^2} + \frac{3}{R^2}\right)(2\,\varphi \,+\, 14\, C) \,=\, 0 \\ &\Box\, C \,-\, C\left(\frac{7}{R_{AdS}^2} + \frac{9}{R^2}\right) \,-\, \frac{1}{2}\,\varphi\left(\frac{1}{R_{AdS}^2} + \frac{3}{R^2}\right) \,=\, 0 \end{aligned}$$

$AdS_3 \times S^7$ (& $AdS_7 \times S^3$) Vacua

- $\ell \neq 0$:

$$R^2_{AdS}\,\mathcal{M}^2 \;=\; \frac{L_3}{3}\,(\sigma_3-1)\,\mathbf{1}_3 + \begin{pmatrix} 4+3\sigma_3 & -\frac{7}{2}\,\sigma_3 & \frac{L_3}{2}\,(\sigma_3-1) \\ -2\,\sigma_3 & 2\,\sigma_3+\tau_3 & -\frac{L_3}{3}\,(\sigma_3-1) \\ 8\,\sigma_3 & -4\,\sigma_3 & 0 \end{pmatrix},$$

$$\sigma_3 \to \frac{3}{2}\,, \qquad \tau_3 \to \frac{9}{2}\,, \qquad L_3 \to \ell(\ell+6)$$

$$\mathcal{M}^2\,R^2_{AdS} \;\geq\; -\,1\ ?$$

- Breitenlohner-Freedman Bound Violated for ℓ= 2,3,4 :

(Gubser, Mitra, 2002)

- [Similar result for heterotic SO(16)xSO(16) for ℓ= 1]

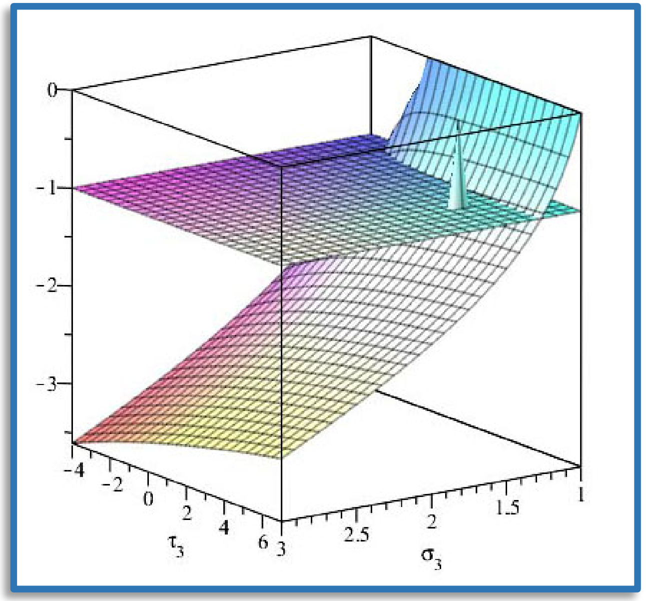
$$\frac{\ell(\ell+6)}{6}+4$$
$$\frac{(\ell+6)(\ell+12)}{6}$$
$$\frac{\ell(\ell-6)}{6}$$

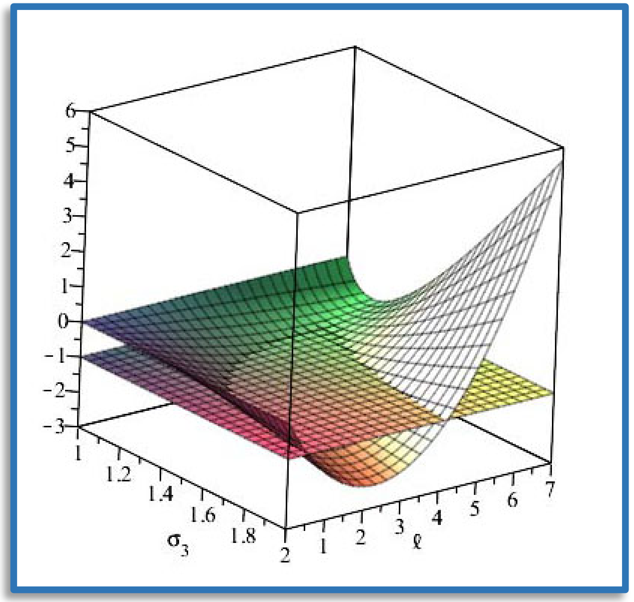
$AdS_3 \times S^7$ (& $AdS_7 \times S^3$) Vacua

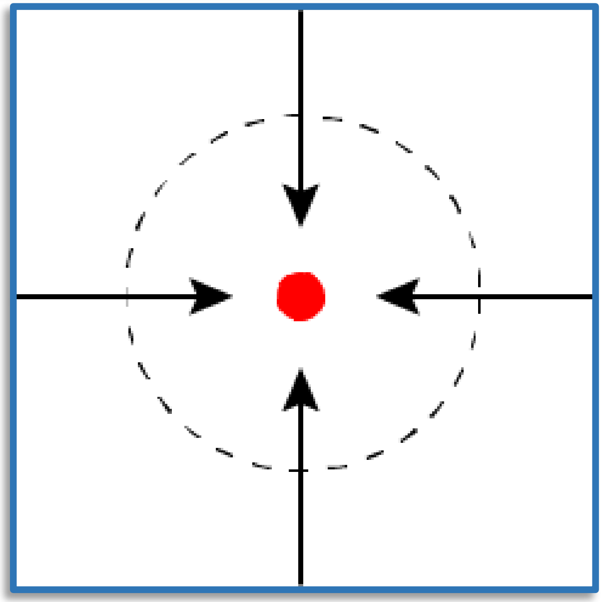
❖ $AdS_3 \times S^7$ Orientifold Flux vacua: WEAK coupling but UNSTABLE

- Violations of Breitenlohner-Freedman bound for scalar modes
- HOWEVER: wide nearby regions of stability. From QUANTUM CORRECTIONS?
- (At least in heterotic): instabilities can be removed by simple internal projections
- STILL: Futher non-perturbative instabilities (brane decays)

(Antonelli, Basile, 2019)







Akin to Electro- (or Gravito-) static Instabilities ?

Dudas-Mourad Vacua

(Basile, Mourad, AS, 2018)

❖ Dudas-Mourad vacua: STABLE but STRONG COUPLING(s)

$$ds^2 \;=\; e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu \;+\; 2\,dx^\mu\,dz\,\partial_\mu D \;+\; (1+C)\,dz^2\right] \;,$$

- BUT: D can be gauged away, and then C = - 7 A (looks familiar from Cosmology …)

$$A'' + A'\left(24\,\Omega' \;-\; \frac{2}{\phi'}\;e^{2\Omega}\,V_\phi\right) \;+\; A\left(m^2 \;-\; \frac{7}{4}\;e^{2\Omega}\,V \;-\; 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) \;=\; 0$$

- Can turn into a Schrödinger-like form (recall the preceding BF arguments):

$$m^2\,\Psi \;=\; \left(b \;+\; \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$
$$b \;=\; \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1\;+\;\frac{9}{4}\,\alpha_O\,y^2} \;>\; 0$$

- NO tachyons in 9D → STABILITY

The Climbing Scalar

(Basile, Mourad, AS, 2018)

- COSMOLOGY : the issue is the time evolution of perturbations

- For large η V is negligible and tensor perturbations evolve as

$$
\begin{aligned}
h''_{ij} \quad &+ \quad \frac{1}{\eta}\, h'_{ij} \;+\; \mathbf{k}^2\, h_{ij} \;=\; 0 \\
h_{ij} \quad &\sim \quad A_{ij}\, J_0(k\eta) \;+\; B_{ij}\, Y_0(k\eta) \qquad (\mathbf{k} \neq 0) \\
h_{ij} \quad &\sim \quad A_{ij} \;+\; B_{ij}\; \log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)
\end{aligned}
$$

- NOTICE: logarithmic growth for k=0 (instability of isotropy) !!

The Climbing Scalar

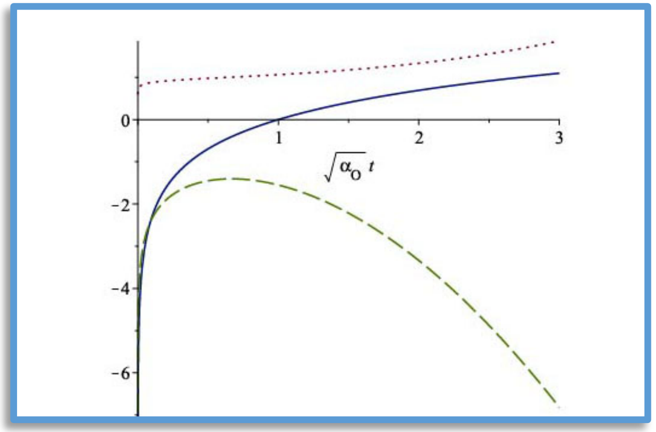
- More in detail:

$$\begin{aligned}
\phi &= -\frac{3}{4}\,\alpha_O t^2 + \frac{1}{3}\ln\left|\alpha_O t^2\right| + \Phi_0 \,, \\
ds_O^2 &= -\,e^{-\frac{3}{2}\Phi_0}\left|\alpha_O t^2\right|^{-\frac{1}{2}} e^{\frac{9}{8}\alpha_O t^2} dt^2 + \left|\alpha_O\, t^2\right|^{\frac{1}{18}} e^{\alpha_O \frac{t^2}{8}}\, d\mathbf{x}\cdot d\mathbf{x} \\
ds_O^2 &= e^{2\,\Omega(\eta)}\left(-\,d\eta^2 + d\mathbf{x}\cdot d\mathbf{x}\right)\,, \\
\phi &= \phi(\eta) \\
d\eta &= \left|\sqrt{\alpha_O}\,t\right|^{-\frac{5}{9}} e^{-\frac{3}{4}\Phi_0}\, e^{\frac{\alpha_O}{2}\,t^2}\, dt
\end{aligned}$$

- Exact solution in terms of the "parametric time" t:

  (' = derivative w.r.t. conformal time η)

$$\begin{aligned}
h''_{ij} &+ 8\,\Omega'\,h'_{ij}\ \left[+\,\mathbf{k}^2\,h_{ij}\right] = 0 \\
h_{ij} &= A_{ij} + B_{ij}\,\log\left(\sqrt{\alpha_O}t\right)
\end{aligned}$$



The Climbing Scalar

Cosmological Models Behave BETTER

a) CLIMBING SCALAR: INSTABILITY of ISOTROPY (k=0 only)

b) STABLE in (D < 10) DESCENT can be driven by mild (brane-induced) potential *[Lucchin-Matarrese attractor]*

Perfect Fluid Picture:

$$\frac{p}{\rho} = \alpha = \frac{T - V}{T + V}$$

$$ds^2 = \left(\frac{\eta}{\eta_0}\right)^{\frac{4}{(d-1)(1+\alpha) - 2}} \left[- d\eta^2 + d\mathbf{x} \cdot d\mathbf{x}\right]$$

$$h_{ij} = A_{ij} + B_{ij}\, \eta^{\frac{(d-1)(\alpha-1)}{(d-1)(\alpha+1)-2}}$$

❖ COMPACTIFICATION to D < 10 ?

Solutions with Flux and Tension

(Mourad, AS, to appear)

General Lesson:

a) SPATIAL PROFILES: finite intervals ←→ strong coupling at one or both ends [ALSO: Scherk-Schwarz-like extensions in intervals]

a) COSMOLOGY: better behavior (climbing), even with initial anisotropy

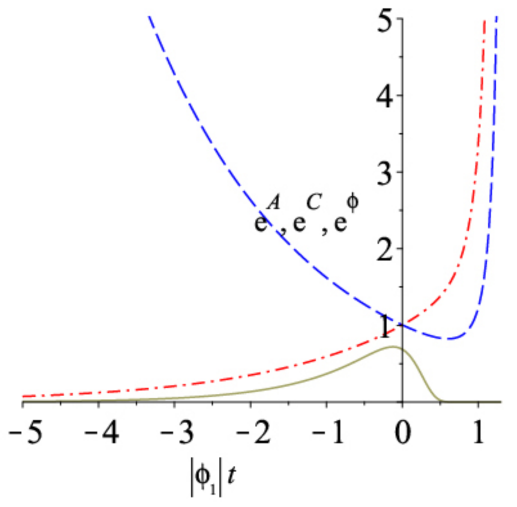
b) FLUXES: can remove SOME singularities induced by the tension T

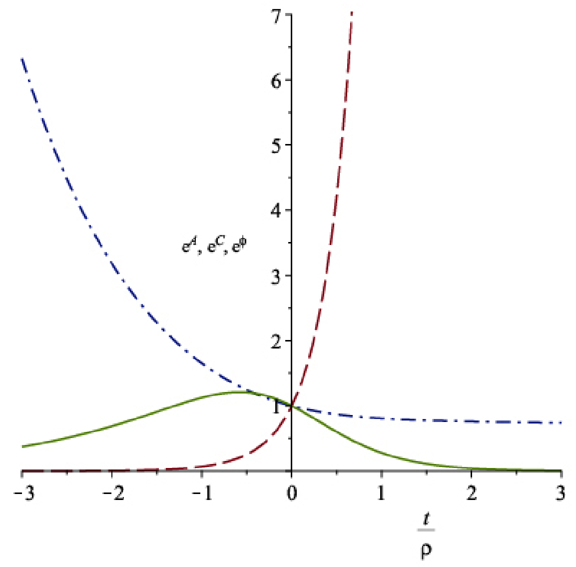
$$V = V_0 \, e^{2\gamma\varphi} \rightarrow_{D=10} \; V_0 \, e^{\frac{3}{2}\gamma\phi}$$

Anisotropic Cosmologies:

a) LEFT: $\gamma \leq \gamma_c$

b) RIGHT: $\gamma > \gamma_c$





Best Wishes, Bernard !