Black Holes and Scherk-Schwarz

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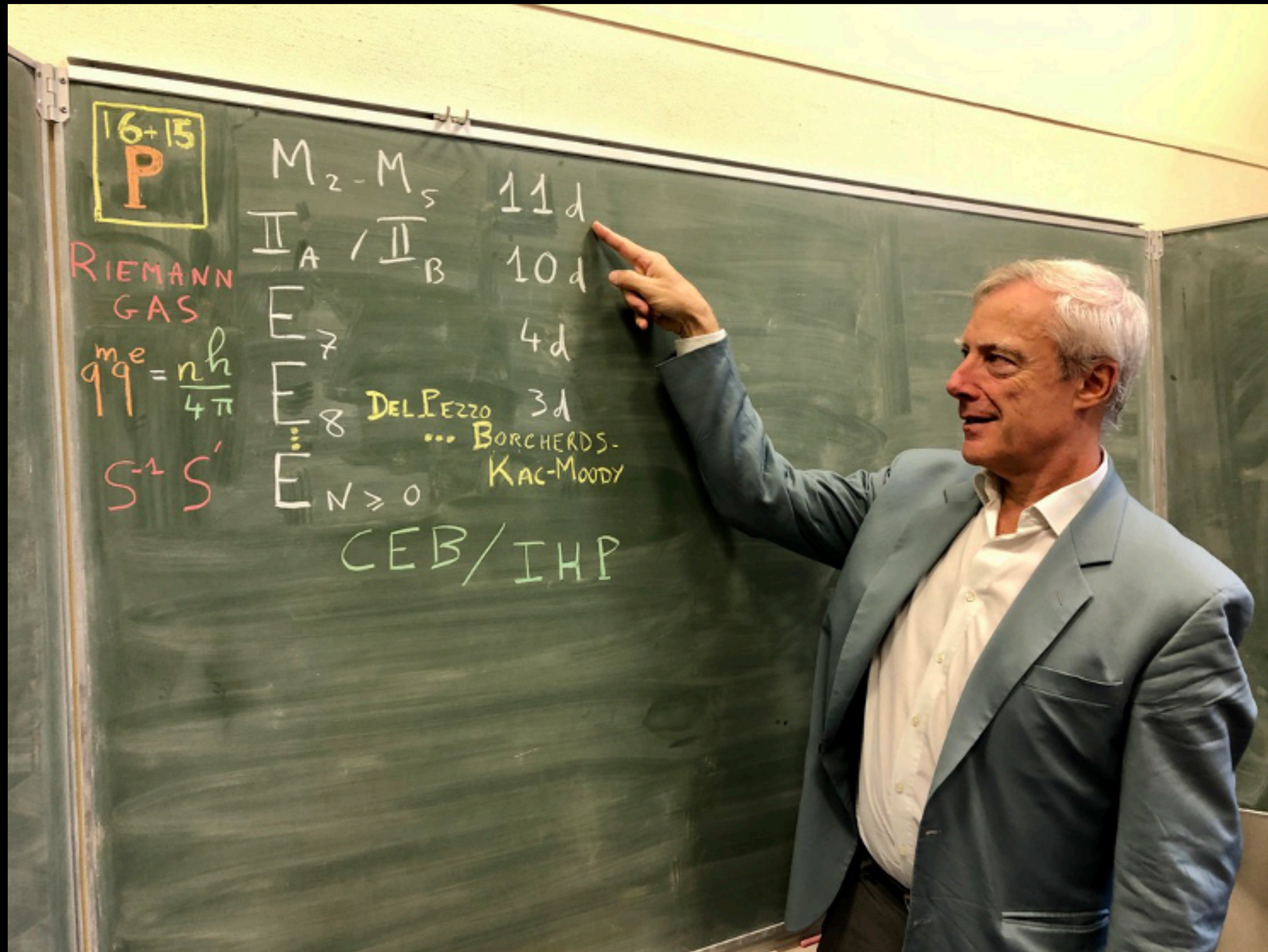
1912.xxxxx - Chris Hull, Eric Marcus, Koen Stemerdink, S.V.

[Earlier related work: Gaddam, Gnecchi, S.V., Varela, ’14]



Dear Bernard,…

Cremmer, Julia and Scherk

Intro & Motivation

- We have used string theory to derive the Bekenstein-Hawking area law for black holes (Strominger, Vafa ’96,…)
- These Black Holes are supersymmetric…
- And the string theories also have (high degree of) supersymmetry…
- Can we (partially) break supersymmetry?

Intro & Motivation

BPS Black Holes a la Strominger and Vafa ('96)

IIB D=10		D1-D5
↓	T^4	↓
(2,2) D=6		BPS Black String
↓	S^1	↓
N=8 D=5		BPS Black Hole

Intro & Motivation

BPS Black Holes a la Strominger and Vafa ('96)

D1-D5		(4,4) CFT
↓	T^4	↓
BPS Black String		$AdS_3 \times S^3$
↓	S^1	↓
BPS Black Hole		$AdS_2 \times S^3$

$$S = 2\pi\sqrt{N_1 N_5 N_k}$$

New Set Up

BPS Black Holes a la Strominger and Vafa ('96)

	IIB D=10		D1-D5	
	↓	T^4	↓	
	(2,2) D=6		BPS Black String	
Duality Twist/ Scherk-Schwarz	↓	S^1	↓	**Duality Twist/ Scherk-Schwarz**
	$N<8$ D=5		New Black Hole	

$$S = \frac{1}{4}A = ???$$

Scherk-Schwarz

- D+1 dimensional supergravity with symmetry group G

$$\psi \to g\psi \quad g \in G$$

- SS ansatz: $\psi(x^{\mu}, z) = \exp(Mz)\psi(x^{\mu})$
- $M \in \mathfrak{g}$ is called the mass matrix
- Monodromy: $\mathcal{M} = \exp(M)$
- Results in gauged U(1) supergravity in D dimensions

Scherk and Schwarz '79, Cremmer, Scherk and Schwarz '79,…

Scherk-Schwarz and Duality Twists

- We can twist with a general element of the U-duality group G (classified by conjugacy classes)
- Choosing inside the T-duality group, the theory at the minimum has an exact CFT description as an orbifold
- In string theory, quantization condition $G(\mathbb{R}) \to G(\mathbb{Z})$
- Twisting in the R-symmetry (partially) breaks supersymmetry

…, Dabholkar and Hull, '02

(Partial) Susy Breaking

D=6 N=8 $\frac{G}{H} = \frac{SO(5,5)}{SO(5)\times SO(5)}$

Kaluza Klein

Duality Twist/ Scherk-Schwarz

D=5 N=8 N=6 N=4 N=2 N=0

R-symmetry: Sp(4) Sp(3) Sp(2) Sp(1)

$$SO(5) \simeq USp(4) \quad Sp(n) = USp(2n)$$

6D to 5D Scherk-Schwarz

Twist in R-symmetry

$$SO(5)_L \times SO(5)_R \simeq USp(4)_L \times USp(4)_R$$

to break supersymmetry

$$USp(4) \rightarrow USp(2) \times USp(2)$$

Twist/mass matrix

$$M_{\mathrm{L}}^{\mathfrak{usp}(4)} = \begin{pmatrix} 0 & 0 & -m_1 & 0 \\ 0 & 0 & 0 & -m_2 \\ m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \end{pmatrix}, \qquad M_{\mathrm{R}}^{\mathfrak{usp}(4)} = \begin{pmatrix} 0 & 0 & -m_3 & 0 \\ 0 & 0 & 0 & -m_4 \\ m_3 & 0 & 0 & 0 \\ 0 & m_4 & 0 & 0 \end{pmatrix}.$$

Fields	Representation	Masses
Scalars	$(\mathbf{5},\mathbf{5})$	$\|\pm m_1 \pm m_2 \pm m_3 \pm m_4\|$
		$\|\pm m_1 \pm m_2\|$
		$\|\pm m_3 \pm m_4\|$
		0
Vectors	$(\mathbf{4},\mathbf{4})$	$\|\pm m_{1,2} \pm m_{3,4}\|$
Tensors	$(\mathbf{5},\mathbf{1})$	$\|\pm m_1 \pm m_2\|$, 0
	$(\mathbf{1},\mathbf{5})$	$\|\pm m_3 \pm m_4\|$, 0
Gravitini	$(\mathbf{4},\mathbf{1})$	$\|\pm m_{1,2}\|$
	$(\mathbf{1},\mathbf{4})$	$\|\pm m_{3,4}\|$
Dilatini	$(\mathbf{5},\mathbf{4})$	$\|\pm m_1 \pm m_2 \pm m_{3,4}\|$
		$\|\pm m_{3,4}\|$
	$(\mathbf{4},\mathbf{5})$	$\|\pm m_{1,2} \pm m_3 \pm m_4\|$
		$\|\pm m_{1,2}\|$

See also Andrianopoli, Ferrara, Lledo, ’04

Partial SUSY Breaking

Patterns of susy breaking:

$$m_1 = m_2 = m_3 = 0 \rightarrow N = 6$$

$$m_1 = m_2 = 0 \rightarrow N = 4 \ (2,0)$$

$$m_1 = m_3 = 0 \rightarrow N = 4 \ (1,1)$$

$$m_3 = m_4 = 0 \rightarrow N = 4 \ (0,2)$$

$$m_4 = 0 \rightarrow N = 2 \ (0,1)$$

$$m_1 = 0 \rightarrow N = 2 \ (1,0)$$

The three N=4 theories have the same massless spectrum, but different massive multiplets. (2,0) and (0,2) are related by parity, but parity is anomalous (more later!) Similarly for N=2.

Black Holes

D = 10 : D1-D5-P

↓ T^4

D = 6 : Black String

Charged by:
- 2-form
- scalar

↓ S^1

D = 5 : Black Hole

Charged by:
- 3 vectors
- 2 scalars

$$M_{\mathrm{L}} = \begin{pmatrix} 0 & -(m_1+m_2) & 0 & 0 & 0 \\ m_1+m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(m_1-m_2) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1-m_2 & 0 & 0 \end{pmatrix},$$

$$M_{\mathrm{R}} = \begin{pmatrix} 0 & -(m_3+m_4) & 0 & 0 & 0 \\ m_3+m_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(m_3-m_4) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3-m_4 & 0 & 0 \end{pmatrix}.$$

Black Hole fields remain massless.This twist matrix does not belong to T-duality O(4,4)

Black Holes

D = 10 : F1-NS5-P

↓ T^4

D = 6 : Black String

Charged by:
- 2-form
- scalar

↓ S^1

D = 5 : Black Hole

Charged by:
- 3 vectors
- 2 scalars

$$M_{\rm L} = \begin{pmatrix} 0 & -(m_1+m_2) & 0 & 0 & 0 \\ m_1+m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(m_1-m_2) & 0 \\ 0 & 0 & m_1-m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This twist matrix is conjugate to the previous one but DOES belong to T-duality group: hence string description.

Full string description

Two special cases:

$$m_1 = m_2 = 0 : \; N = 4 \; (0,2) \; and \; m_3 = m_4$$
$$m_3 = m_4 = 0 : \; N = 4 \; (2,0) \; and \; m_1 = m_2$$

Twist matrix now is part of T-duality group. These string theories can be described by asymmetric orbifolds. Similarly for all mass parameters equal, but this leads to (0,0). Perhaps this is a very interesting theory!

Both preserve F1-F5 and D1-D5 black hole system.

Entropy and Chern-Simons

- Black hole entropy without susy breaking

$$S_{\rm BH} = \frac{A}{4G_N^{(5)}} = 2\pi\sqrt{N_1 N_5 N_K}$$

- Now integrate out chiral and self-dual fields. This yields additional corrections to Chern-Simons terms [..., Bonetti, Grimm, Hohenegger, '13]

$$\mathcal{L}_{AFF} = -\frac{1}{6}\, k_{AFF}\, A_1 \wedge F_2 \wedge F_2\,, \qquad \mathcal{L}_{ARR} = -\frac{1}{2}\, k_{ARR}\, A_1 \wedge \mathrm{Tr}\,(R_2 \wedge R_2)$$

Entropy and Chern-Simons

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Explicit one-loop calculations using sugra techniques:

- No contributions for N = 8, 6 (supersymmetry)
- N = 4:

$$k_{AFF} = 0$$

$$k_{ARR}^{(2,0)} = | k_{ARR}^{(2,0)} |$$

$$k_{ARR}^{(1,1)} = 0$$

$$k_{ARR}^{(0,2)} = -k_{ARR}^{(2,0)}$$

Entropy and Chern-Simons

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We have expressions for N=2 as well in which kAFF is nonzero. String interpretation not clear though.

Entropy and Chern-Simons

The additional Chern-Simons terms affect the black hole entropy [Castro, Davis, Kraus, Larsen, '07]

$$S_{\rm BH} \;=\; 2\pi \sqrt{N_1 N_5 \hat{N}_K \,\frac{2\left(1+\sqrt{1+4\pi^2\, k_{AFF}\,\frac{N_1 N_5}{\hat{N}_K^2}}+\frac{4\pi^2}{3}\,k_{AFF}\,\frac{N_1 N_5}{\hat{N}_K^2}\right)^2}{\left(1+\sqrt{1+4\pi^2\, k_{AFF}\,\frac{N_1 N_5}{\hat{N}_K^2}}\right)^3}}$$

$$\hat{N}_K \;=\; N_K + 12\pi^2\, k_{ARR}$$

Challenge for D1-D5 CFT? For N=4, (positive/negative shift in L_0 ?

String theory

- String theory embedding discussed in Dabholkar and Hull '02.

$$S^{-1}\mathcal{M}\,S \;\in\; \mathrm{SO}(5,5,\mathbb{Z})$$

- Results in four quantised twist parameters, e.g.

$$m_1 \;=\; \tfrac{1}{2}\big(k_1 + k_2 + k_3 + k_4\big) \qquad k_i \in \{0, \tfrac{2\pi}{6}, \tfrac{2\pi}{4}, \tfrac{2\pi}{3}\}$$

- Quantised Chern-Simons coefficients, e.g. N = 4 (2,0):

$$k_{ARR} \;\in\; \big\{2, 4, 6, 8, 12, 24\big\}$$

Summary

- IIB on four-torus with SO(5,5) symmetry. Reduce further on S^1.
- Duality twist to 5D such that black hole solutions are preserved.
- Entropy of black holes depend on twist.
- String theory: quantises twist parameters, masses and Chern-Simons coefficients.

Outlook

- What happens to the (4,4) CFT of Strominger-Vafa?
- What are the susy breaking patterns on the CFT?
- (4,2), (2,2), (4,0), (0,4), etc..?
- Calculation of the central charges and Cardy entropy formula?

Many Best Wishes Bernard…

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PS I still don’t eat cheese…

6D to 5D Scherk-Schwarz

- IIB on four-torus: 6D (2,2) supergravity with U-duality group SO(5,5). T-duality group O(4,4)
- Write in U-duality invariant manner [Cremmer, Julia, Lu, Pope, '97]
- Scalar example: 25 scalars in 10 x 10 matrix $\mathcal{H}$

$$\mathcal{H} \to U\,\mathcal{H}\,U^T$$

$$\mathcal{H}(x^\mu, z) = e^{Mz}\,\mathcal{H}(x^\mu)\,e^{M^T z}$$

- Results in massive, charged fields

6D to 5D Scherk-Schwarz

$$e_{(6)}^{-1}\,\mathcal{L}_{\rm s} = \frac{1}{8}\,{\rm Tr}\big[\partial_{\hat\mu}\mathcal{H}^{-1}\partial^{\hat\mu}\mathcal{H}\big]$$

$\downarrow$ S^1

$$e_{(5)}^{-1}\,\mathcal{L}_{\rm s} = \frac{1}{8}\,{\rm Tr}\big[D_\mu\mathcal{H}^{-1}D^\mu\mathcal{H}\big] - V(\mathcal{H})$$

$$D_\mu\mathcal{H} = \partial_\mu\mathcal{H} - \mathcal{A}_\mu^5\left(\mathbb{M}\,\mathcal{H} + \mathcal{H}\,\mathbb{M}^T\right)$$

$$V \propto {\rm Tr}\big[\mathbb{M}^2 + \mathbb{M}^T\mathcal{H}^{-1}\mathbb{M}\,\mathcal{H}\big]$$